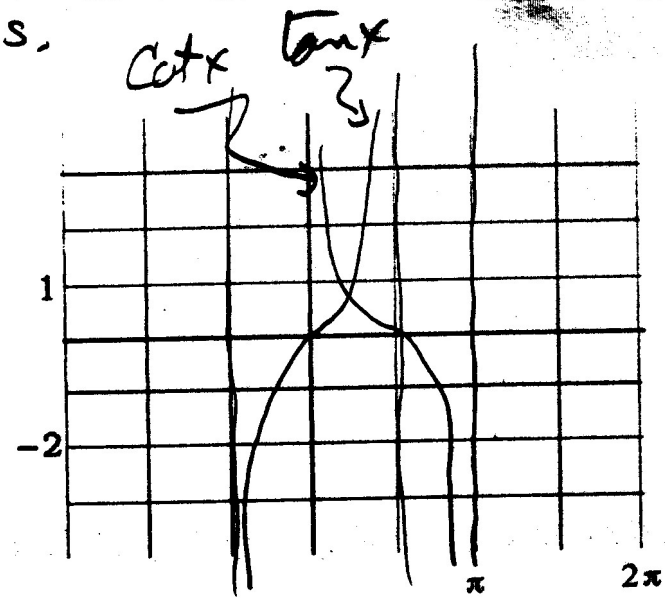
Prove all cps.

II Draw F(x) = Tan x and G(x) = Cot x on the graph supplied. From your picture, determine what must be done TO Tan x to make it "line up" with Cotangent; then show that your changes to Tangent would yield Cotangent. (15 pts)



$-\tan\left(x-\frac{\pi}{2}\right) = ?$

$$\frac{-\sin x \overset{0}{\cos\frac{\pi}{2}} + \sin\frac{\pi}{2}\cos x}{\cos x \underset{0}{\cos\frac{\pi}{2}} + \sin x \sin\frac{\pi}{2}}$$

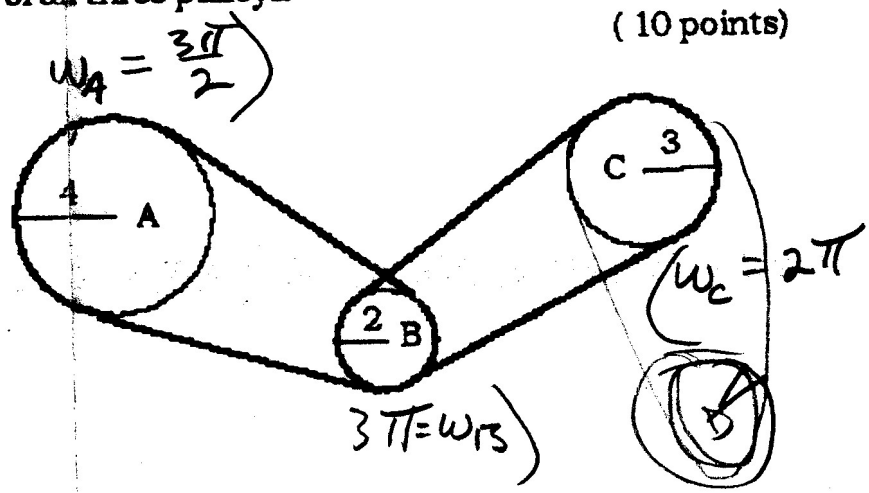
$$\frac{\cos x}{\sin x}$$

$\cot x$

Make it negative and move right $\frac{\pi}{2}$

III Given the three connected pulleys as shown. Pulley "A" is driving at 3 revolutions per 4 seconds in a counter clockwise direction. Pulley "B" has two slots to accomodate both belts. Determine the rotational velocities of all three pulleys.

(10 points)



3 Rev = 4 Sec

1 Rev = $\frac{4}{3}$ Sec. = T_4

$\frac{2\pi}{\frac{4}{3}} = \frac{6\pi}{4} = \frac{3\pi}{2} = \omega_4$

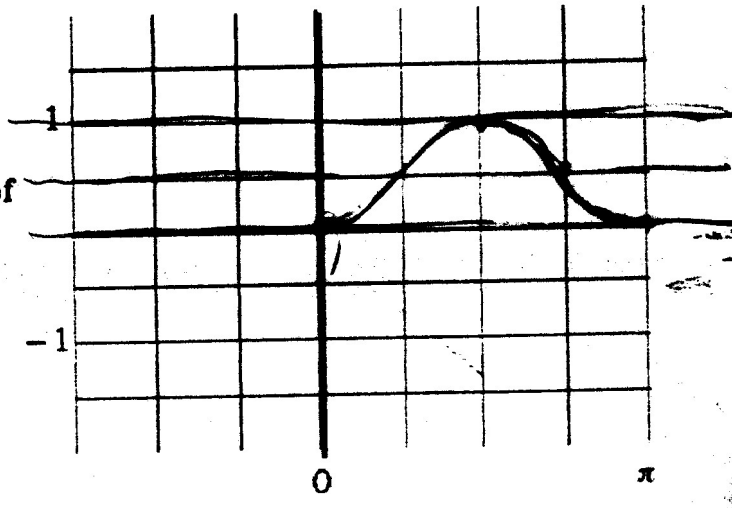
$6\pi = R\omega$ for All wheels

$6\pi = 2\omega_B$ $\quad$ $6\pi = 3\omega_C$

$3\pi = \omega_B$ $\quad$ $2\pi = \omega_C$

EXTRA CREDIT ------- 5 pts -----

On the graph provided, sketch a VERYaccurate graph of



$F(x) = \sin^2 x$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 1 - 2\sin^2 x$

$2\sin^2 x = 1 - \cos 2x$

(Hint: Cos 2x = Cos(x + x))

$\sin^2 x = -\frac{1}{2}\cos 2x + \frac{1}{2}$

Honors Trigonometry Name _____

Per _____ Date _____

Convince *me* that **you** understand the concept.

CHAPTER #3

I This section covers all of the first two pages of this exam. For *each* problem, supply **domain, range, amplitude, period** and **phase shift (including direction word)** and **draw** the graph *in the areas provided*. Be sure your drawing crosses or approaches the Y-axis. You may do the work for this exam *directly on this* exam paper rather than on the "newsprint" as usual. If you do work on newsprint be sure it is labeled with its problem number. Remember, your graph *must* be on the axis provided. **Indicate significant coordinates. You may use decimals for you coordinate responses** (however you will probably spend more time using decimals than in not using decimals). Each problem in this section is worth 15 points.

A) $A(x) = 2\sin\left(\frac{3}{4}x - \frac{3\pi}{16}\right) + 1$

$\frac{2\pi}{\frac{3}{4}} = \frac{8\pi}{3}$ $\quad \frac{3}{4}\left(x - \frac{\pi}{4}\right) = k\pi$ $\quad x - \frac{\pi}{4} = \frac{4k\pi}{3}$ $\quad x = \frac{(16k+3)\pi}{12}$

$k=0 \ \frac{\pi}{4}$ $\quad k=1 \ \frac{19\pi}{12}$ $\quad k=2 \ \frac{35\pi}{12}$ $\quad k=-1 \ -\frac{13\pi}{12}$

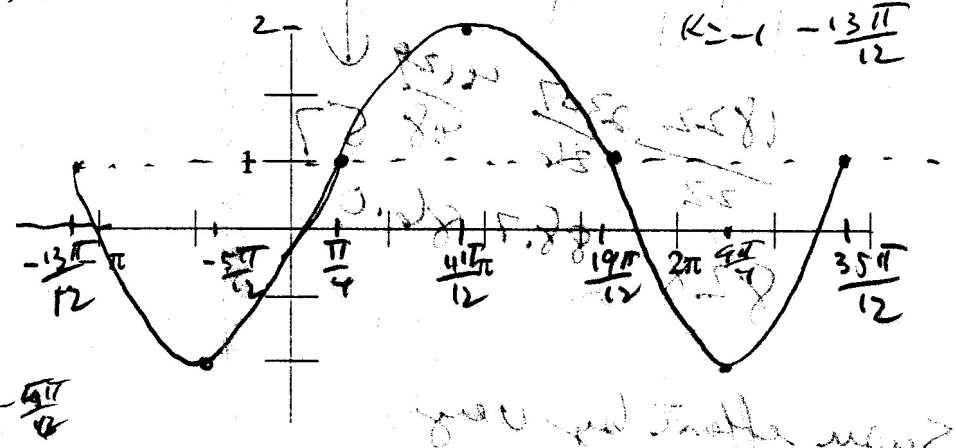
D_A $x \in \mathbb{R}$ $\quad 2\sin\frac{3}{4}\left(x - \frac{\pi}{4}\right) + 1$

R_A $-1 \le A(x) \le 3$

Per $\frac{8\pi}{3}$

Amp 2

P.S. $\frac{\pi}{4}$ right



$\frac{27\pi}{12}$ $\quad \frac{9\pi}{4}$ $\quad -\frac{9\pi}{4}$

B) $B(x) = -\cos\left(\frac{2}{3}x + \frac{2\pi}{9}\right)$

$\frac{2\pi}{\frac{2}{3}} = 3\pi$ $\quad \frac{2}{3}\left(x + \frac{\pi}{3}\right) = \frac{(2k+1)\pi}{2}$ $\quad x - \frac{\pi}{3} = \left(k\pi + \frac{\pi}{2}\right)\frac{3}{2}$ $\quad x = \frac{(18k+5)\pi}{12}$

$-\cos\frac{2}{3}\left(x + \frac{\pi}{3}\right)$

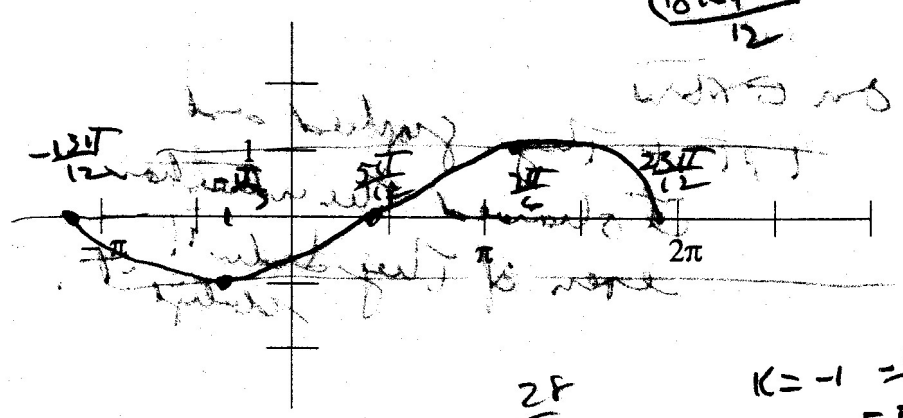
D_B $x \in \mathbb{R}$

R_B $-1 \le B(x) \le 1$

Per 3π

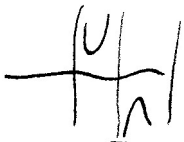
Amp 1 (or −1)

P.S. $\frac{\pi}{3}$ left.



$\frac{28}{12}$

$k=-1 \ \frac{-13\pi}{12}$ $\quad k=0 \ \frac{5\pi}{12}$ $\quad k=1 \ \frac{23\pi}{12}$



C) $C(x) = \csc\left(2x + \frac{\pi}{4}\right) + 1$

D_C $x \neq \frac{(4k-1)\pi}{8}$

R_C $C(x) \leq 0$ or $C(x) \geq 2$

Per π

Amp 1

P.S. $\frac{\pi}{8}$ Left

$2\left(x + \frac{\pi}{8}\right) \overset{\text{set}}{=} k\pi$

$x + \frac{\pi}{8} = \frac{k\pi}{2}$

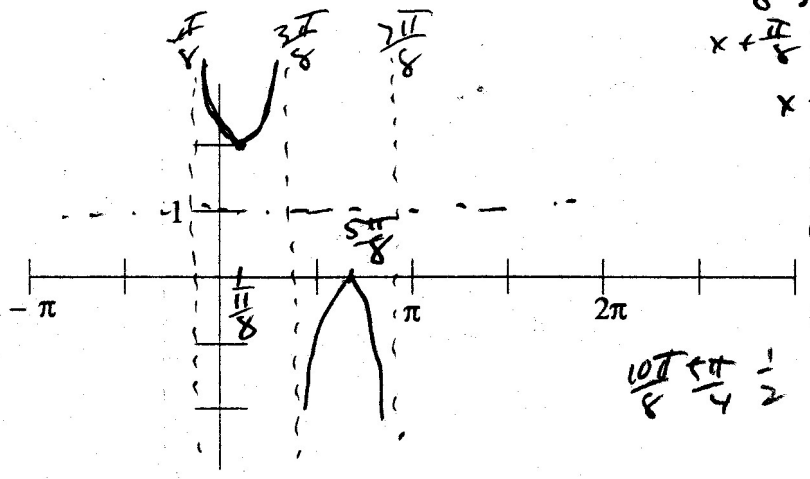
$x = \frac{(4k-1)\pi}{8}$

$k=0$ $-\frac{\pi}{8}$

$k=1$ $\frac{3\pi}{8}$

$k=2$ $\frac{7\pi}{8}$

$\frac{10\pi}{8} \quad \frac{4\pi}{4} \quad \frac{1}{2}$



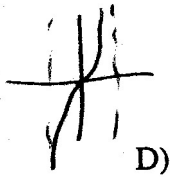
$\frac{\pi}{\frac{4}{3}} = \frac{3\pi}{4}$

$\frac{4}{3}x \overset{\text{set}}{=} \frac{(2k+1)\pi}{2}$

$x = \frac{(6k+3)\pi}{8}$

$k=-1$ $-\frac{3\pi}{8}$

$k=0$ $\frac{3\pi}{8}$



D) $D(x) = \tan\frac{4}{3}x$

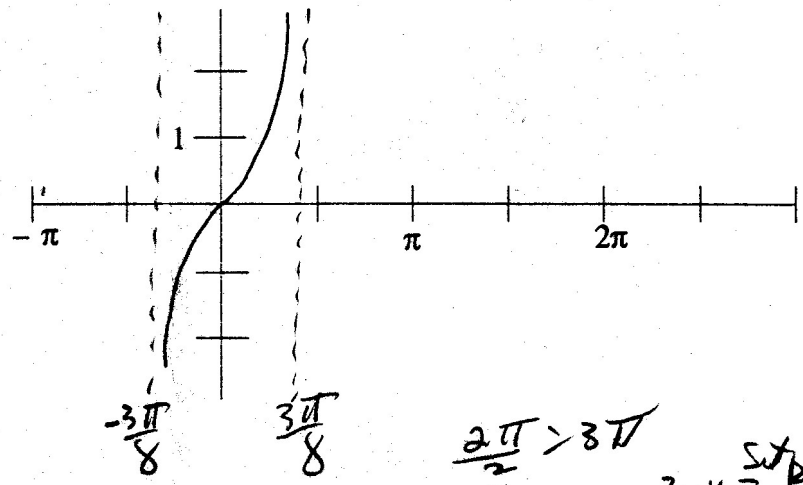
D_D $x \neq \frac{(6k+3)\pi}{8}$

R_D $D(x) \in \mathbb{R}$

Per $\frac{3\pi}{4}$

Amp 1

P.S. None



E) $E(x) = \sec\frac{2}{3}x$

D_E $x \neq \left\{\frac{(6k+3)\pi}{4}\right\}$

R_E $E(x) \leq -1$ or $E(x) \geq 1$

Per 3π

Amp 1

P.S. None

$\frac{2\pi}{\frac{2}{3}} = 3\pi$

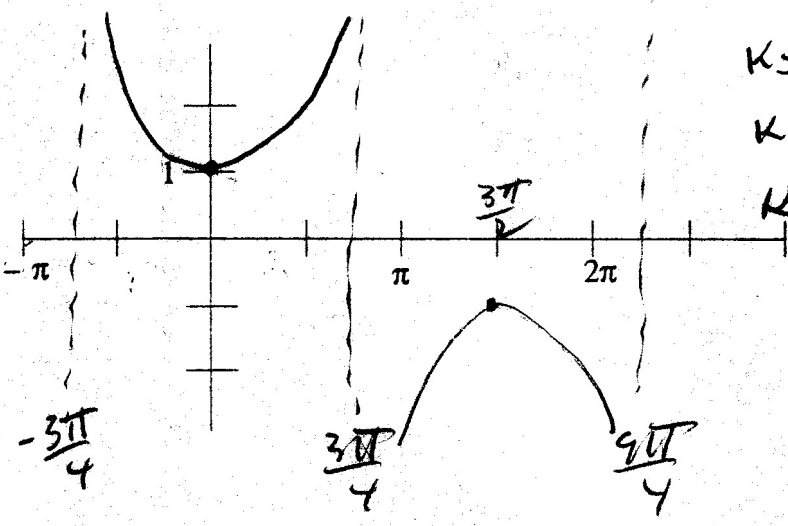
$\frac{2}{3}x \overset{\text{set}}{=} \frac{(2k+1)\pi}{2}$

$x = \frac{(6k+3)\pi}{4}$

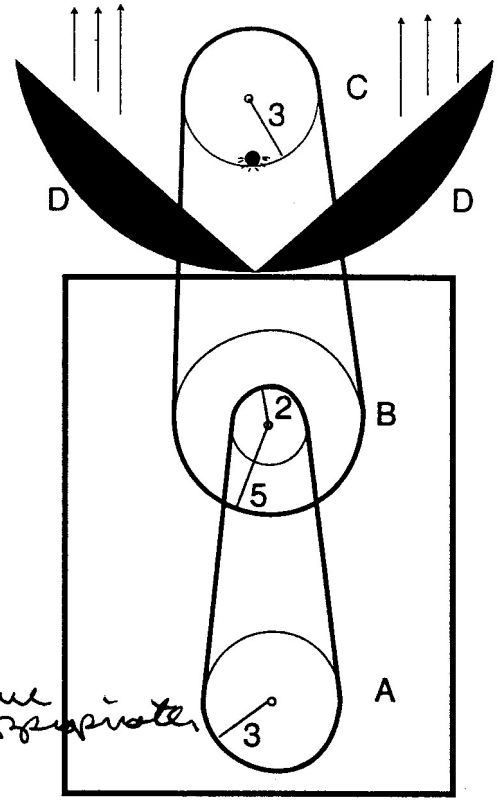
$k=-1$

$k=0$

$k=1$



II The figure on the right is a rotating emergency light. The dot on wheel C is the light source. The black objects labeled D are mirrors which reflect the bright light upward. Wheel A is the motor wheel and it rotates at a constant speed of 15 RPM. Wheel B alters the speed of the rotating light slightly because it is connected to the light wheel by its 5 inch pulley and to the drive wheel by its 2 inch radius pulley. The drive wheel A and the rotating light wheel C each have a radius of 3 inches. (25 pts total)



When the motor is turned on, the red light is at the place indicated in the picture which is as close as it gets to the grey box, 5 inches away.

Your job is to answer the questions below. Include very brief justification for each section. Be sure *I* understand your logic. Your answers should be in ~~seconds rather than minutes.~~ Radians per Second where appropriate

A) Determine the angular velocity in seconds on wheel A.

B) Determine the linear velocity in seconds on wheel A.

C) Determine ***both*** linear velocities in seconds on wheel B. (careful... they *are* different.)

D) Determine the linear velocity in seconds on wheel C.

E) Determine the angular velocity in seconds on wheel C.

F) Write the equation in terms of sine with a positive amplitude which describes the distance the dot on wheel C is from the shaded housing.

G) Draw that graph of your function on the newsprint. The horizontal axis will be seconds and the vertical axis will represent the distance the "dot" is from the grey box. Specifically indicate the period, phase shift, axis movement and amplitude for your function. Indicate significant coordinates (as is usual for a graph).

Extra Credit ..... **5 pts** .....

For certain domain values, the functions $f(x) = \sin x$ and $g(x) = \sin 2x$ from $x = 0$ to $x = 2\pi$ have the same range value. For example, $f(\pi)$ and $g(\pi)$ both have the range value of zero, What are the other domain and range combinations?

II

15 Rev = 60 Sec

1 Rev = 4 Sec = T_A

A) $\omega_A = \frac{2\pi}{4} = \frac{\pi}{2}$

B) $\rho\omega = 3 \cdot \omega_A = \frac{3\pi}{2}$

C) $\frac{3\pi}{2} \overset{\text{set}}{=} 2\omega_B$

$\frac{3\pi}{4} = \omega_B$

$\rho\omega = 5 \cdot \frac{3\pi}{4} = \frac{15\pi}{4}$

The linear velocities are $\frac{3\pi}{2}$ AN $\frac{15\pi}{4}$.

D) $\rho\omega_C = \frac{15\pi}{4}$ because that belt is connected to wheel C.

E) $3\omega_C = \frac{15\pi}{4}$

$\omega_C = \frac{5\pi}{4}$ = coefficient of x

f) $\frac{2\pi}{\frac{5\pi}{4}} = \frac{8}{5}$ = New Period.

ρ = Amplitude = 3

Phase shift = ?

closest = 5

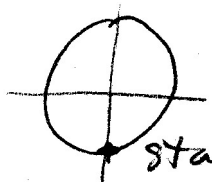
farthest is 11

$\frac{11+5}{2} = 8$

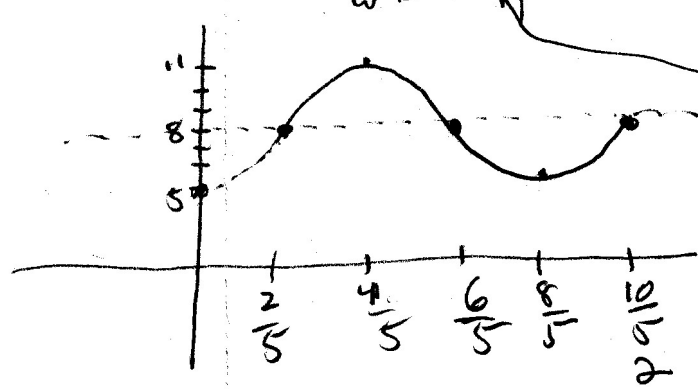
$f(x) = 3 \sin \frac{5\pi}{4}(x - ?) + 8$

$3 \sin\left(\frac{5\pi}{4}x - \frac{\pi}{2}\right) + 8$

$\omega\pi + b$



starts here $-\frac{\pi}{2}$



$\frac{\frac{\pi}{2}}{\frac{5\pi}{4}} = \frac{\pi}{2} \cdot \frac{4}{5\pi} = \frac{2}{5}$

so ? = $\frac{2}{5}$ = Phase shift. To Right.

Extra

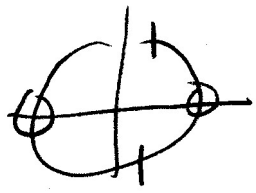
$\sin x \overset{\text{set}}{=} \sin 2x$

$\sin x = 2\sin x \cos x$

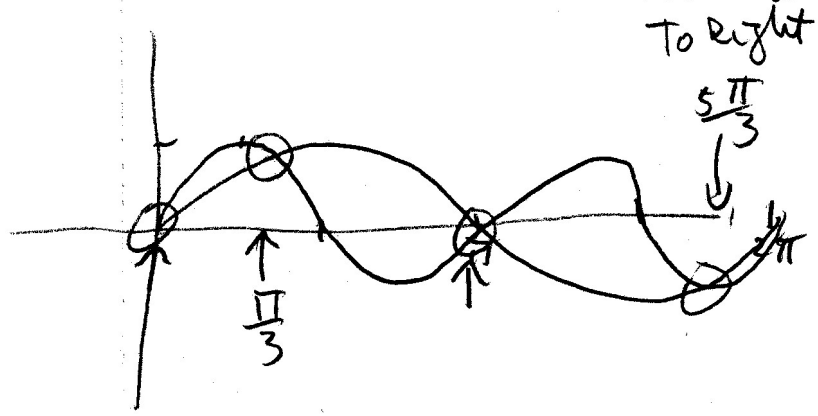
$2\sin x \cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0$ or $\cos x = \frac{1}{2}$



$x \in \{0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}\}$ OR



	4	5	T
100	1	2	
90	2	7	
80	13	13	
70	5	2	
60	0	1	
59	1	1	

101

1822/22 2307/26 4129/48 57

82.8 88.7 86.0

Super effort by very good students on an excellent exam.

one slight wording change on II

on extra credit
lot of they graphed and it showed the interaction even if they didn't realize it.

Honors Trigonometry Name TRG Ch 3

Per _____ Date 2001

Convince *me* that **you** understand the concept.

CHAPTER #3

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A) $A(x) = 2\sin\left(\frac{2}{3}x - \frac{\pi}{6}\right) - 1$

$\frac{2}{3}\left(x - \frac{\pi}{4}\right) \overset{set}{=} k\pi$

$x - \frac{\pi}{4} = \frac{3\pi}{2}$

$x = \frac{(6k+1)\pi}{4}$

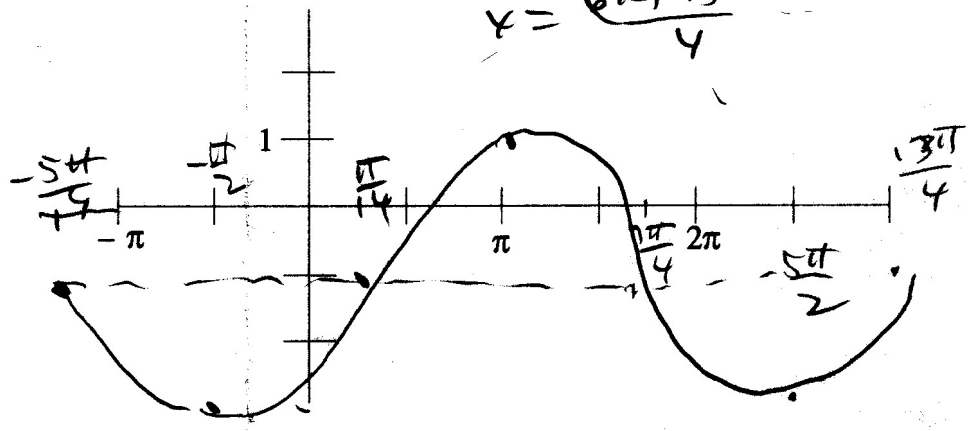
D_A $x \in \mathbb{R}$

R_A $-3 \le A(x) \le 1$

Per 3π

Amp 2

P.S. $\frac{\pi}{4}$ Right



B) $B(x) = -\cos\left(\frac{3}{2}x + \frac{\pi}{2}\right)$

$\frac{3}{2}\left(x + \frac{\pi}{3}\right) \overset{set}{=} \frac{(2k+1)\pi}{2}$

$x = \frac{2k\pi}{3}$

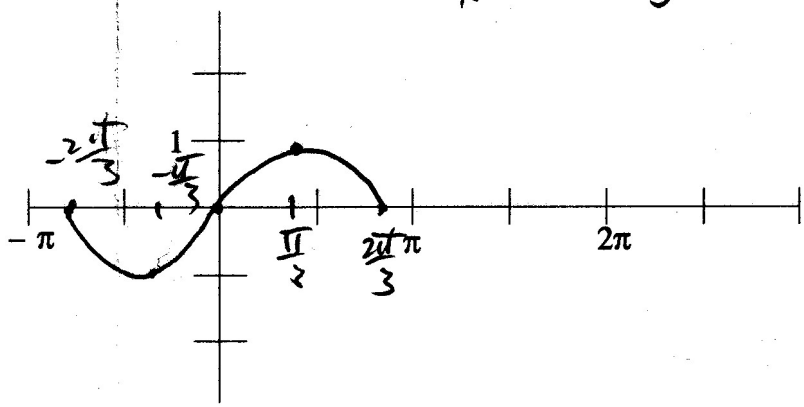
D_B $x \in \mathbb{R}$

R_B $-1 \le B(x) \le 1$

Per $\frac{4\pi}{3}$

Amp 1 or -1

P.S. $\frac{\pi}{3}$ Left



C) $C(x) = \sec\left(x - \frac{2\pi}{3}\right) - 1$

D_C $x \neq \left\{\frac{(6K+7)\pi}{6}\right\}$

R_C $C(x) \leq -2$ or $C(x) \geq 0$

Per 2π

Amp 1

P.S. $\frac{2\pi}{3}$ Right.

$x - \frac{2\pi}{3} \overset{set}{=} \frac{(2K+1)\pi}{2}$

$x = K\pi + \frac{\pi}{2} + \frac{2\pi}{3}$

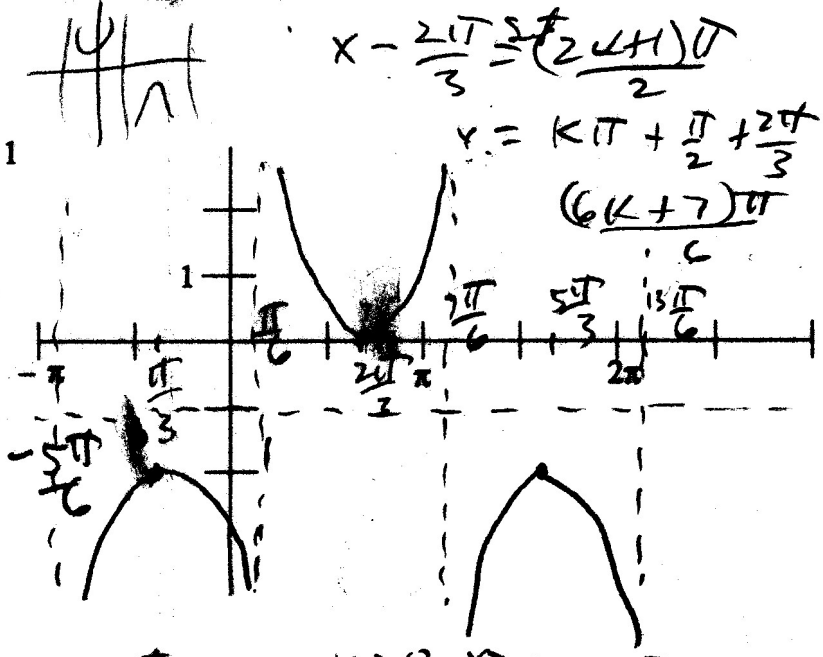
$\frac{(6K+7)\pi}{6}$

$K=-2$ $-\frac{5\pi}{6}$

$K=-1$ $\frac{\pi}{6}$

$K=0$ $\frac{7\pi}{6}$

$K=1$ $\frac{13\pi}{6}$



D) $D(x) = \cot \frac{4}{3} x$

D_D $x \neq \left\{\frac{3K\pi}{4}\right\}$

R_D $D(x) \in \mathbb{R}$

Per $\frac{3\pi}{4}$

Amp 1

P.S. None

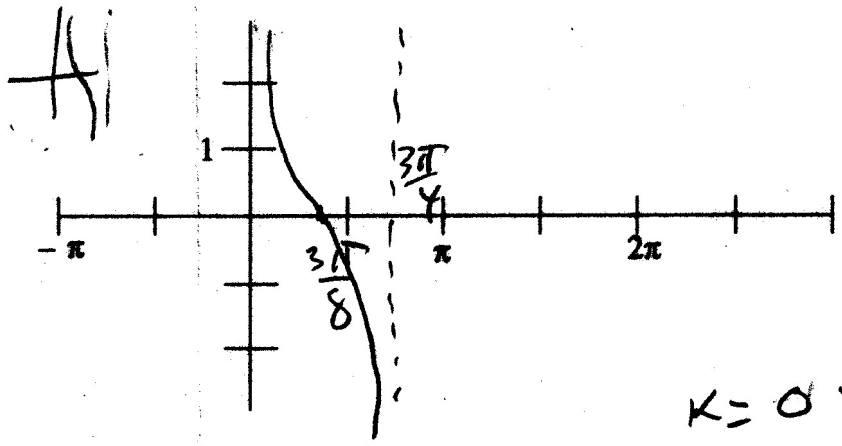
$\frac{4}{3}x \overset{set}{=} K\pi$

$x = \frac{3K\pi}{4}$

$K=0$ $x=0$

$K=1$ $\frac{3\pi}{4}$

$\frac{\pi}{\frac{4}{3}} = \frac{3\pi}{4}$



E) $E(x) = \csc \frac{2}{3} x$

D_E $x \neq \left\{\frac{3K\pi}{2}\right\}$

R_E $E(x) \leq -1$ or $E(x) \geq 1$

Per 3π

Amp 1

P.S. None

$\frac{2}{3}x \overset{set}{=} K\pi$

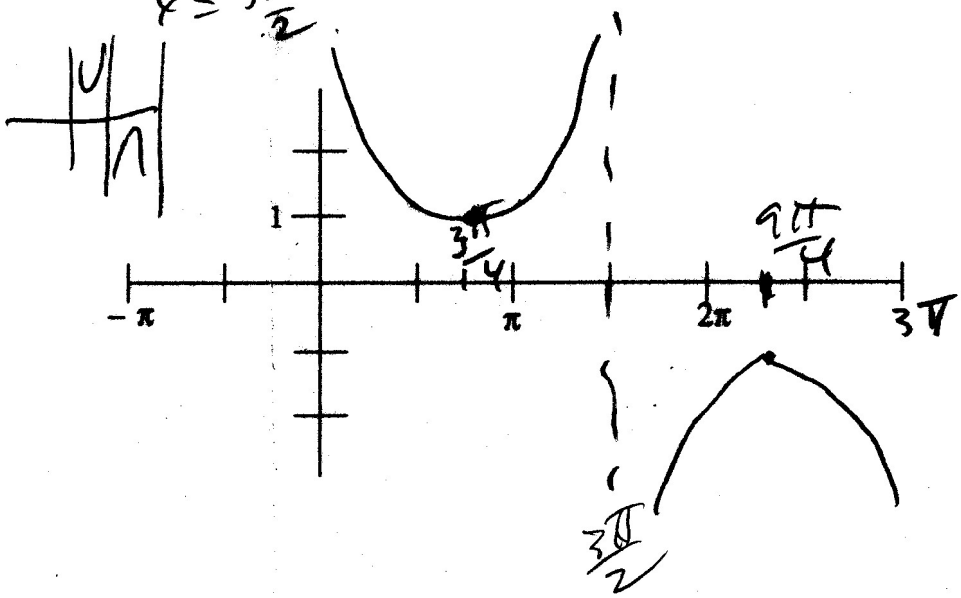
$x = \frac{3K\pi}{2}$

$\frac{2\pi}{\frac{2}{3}} = 3\pi$

$K=0$ $x=0$

$K=1$ $x=\frac{3\pi}{2}$

$K=2$ $x=3\pi$



$T = 24$ ie $dP_1 = dP_2 + 2\pi$

-5

Do the remainder of the exam on the newsprint as usual.

a) $dP_1 = dP_2 + \frac{11\pi}{2}$

$\frac{3\pi}{3} = \frac{3\pi}{4}\tau + \frac{11\pi}{2}$

$\frac{1}{4}\tau = \frac{11}{2}$ $\tau = 22$

c) dP_1 $\rho\omega\tau$

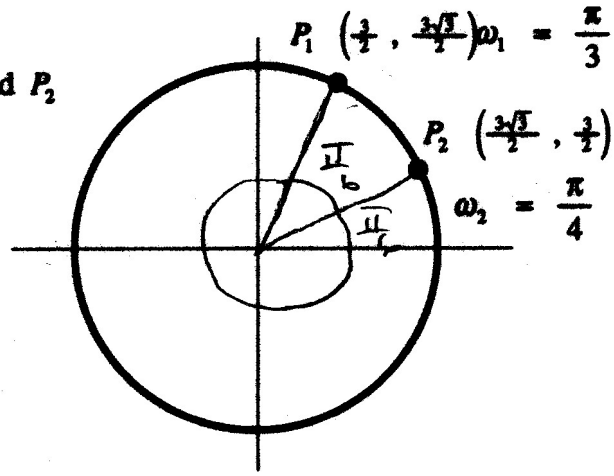
$3 \cdot \frac{\pi}{3} \cdot 22 = 22\pi$

D) dP_2 $3 \cdot \frac{\pi}{\cancel{4}_2} \cdot \cancel{22}^{11} = \frac{33\pi}{2}$

II

(25 pts total)

Given the points P_1 and P_2 rotating around the circle with uniform circular motion. At time $t = 0$, P_1 and P_2 are at the locations indicated on the picture on the right.



a) **When (find t) will P_1 and P_2 occupy the same location on the circle?**

b) **What are the coordinates of that point?**

c) **How far did P_1 travel?**

d) **How far did P_2 travel?**

B) $(\rho\cos(\omega\tau + b), \rho\sin(\omega\tau + b))$

$3\cos\left(\frac{\pi}{3} \cdot 22 + \frac{\pi}{3}\right)$

$\frac{23\pi}{3}$

$\frac{18\pi}{3} + \frac{5\pi}{3}$

$3\cos\left(\frac{5\pi}{3}\right), 3\sin\left(\frac{5\pi}{3}\right)$ same

$\left(\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right)$

$3\cos\frac{\pi}{\cancel{4}_2} \cdot \cancel{22}^{11} + \frac{\pi}{6}$

$\frac{(3 \cdot 3 + 1)\pi}{6}$

$\frac{34\pi}{6} = \frac{17\pi}{3}$

$3\cos\frac{20\pi}{3} + \frac{5\pi}{3}$

$\left(3\cos\frac{5\pi}{3}, 3\sin\frac{5\pi}{3}\right)$

$22\pi - \frac{33\pi}{2} =$

$\frac{(44 - 33)\pi}{2} =$

$\frac{11\pi}{2}$ = dist P_1 had to go ~~[illegible]~~

Extra Credit ..... **5 pts** .....

Given: $f(x) = a\cos(bx + c)$ and
$g(x) = d\cos(ex + f)$

$3\cos\left(\frac{\pi}{3}x + \frac{\pi}{3}\right)$

$\left(\frac{\pi}{4}x + \frac{\pi}{6}\right)$

where $f(x)$ describes the movement of P_1 in section II above and $g(x)$ describes the movement of P_2 in section II above.

Find the explicit values of the constants: a, b, c, d, e, and f.

Explain, of course.

$a = 3$ Radius

$b = \frac{\pi}{3}$ ω_1

$c = \frac{\pi}{3}$ ← headstart (from 3, 0)

$d = 3$ Radius.

$E) = \frac{\pi}{4}$ ω_2

$f) = \frac{\pi}{6}$ ← headstart

Note: The phase shift for f and g

$\frac{\pi}{3}(x+1)$

$\frac{\pi}{4}\left(x + \frac{2}{3}\right)$

f PS 1 Left

g PS $\frac{2}{3}$ Left.

10/31/97

100	6
90	10
80	4
70	3
60	4
59	3

105

35

10/26/01

	3	5
110	1	3
90	5	9
80	11	5
70	6	7
60	1	3
54	2	2

103

45

1939	2334	4273
25	28	53
77.6	83.4	80.6

Excellent, easy, exam.

I change You → Yoan

$\frac{2530}{30}$ 84.3

without bottom 2 (who have never passed an exam)

87.1

Honors Trigonometry Name Trig Ch3

Per _____ Date 2002

Convince *me* that **you** understand the concept.

CHAPTER #3

I This section covers all of the first two pages of this exam. For *each* problem, supply **domain, range, amplitude, period** and **phase shift (including direction word)** and **draw** the graph *in the areas provided*. Be sure your drawing crosses or approaches the Y-axis. You may do the work for this exam *directly on this* exam paper rather than on the "newsprint" as usual. If you do work on newsprint be sure it is labeled with its problem number. **Indicate significant coordinates You *may* use decimals for your coordinate responses** (however you will probably spend more time using decimals than in not using decimals). Each problem in this section is worth 15 points.

A) $A(x) = 2\cos\left(\frac{2}{3}x - \frac{\pi}{6}\right) - 1$

$\frac{1}{2}\left(-\frac{\pi}{2} + \pi\right) = \frac{\pi}{4}$

$\frac{1}{2}\left(\pi + \frac{3\pi}{2}\right) = \frac{2\pi}{4}$

$\frac{2\pi}{\frac{2}{3}} = 3\pi$

$\frac{2}{3}\left(x - \frac{\pi}{4}\right) \overset{\text{Set}}{=} \frac{(2k+1)\pi}{2}$

$x - \frac{\pi}{4} = \frac{3\pi}{4}(2k+1)$

$= \frac{6k\pi}{4} + \frac{3\pi}{4} + \frac{\pi}{4}$

$x = \frac{(3k+2)\pi}{2}$

$k=-1 \quad x = -\frac{\pi}{2}$

$k=0 \quad x = \pi$

$k=1 \quad \frac{5\pi}{2}$

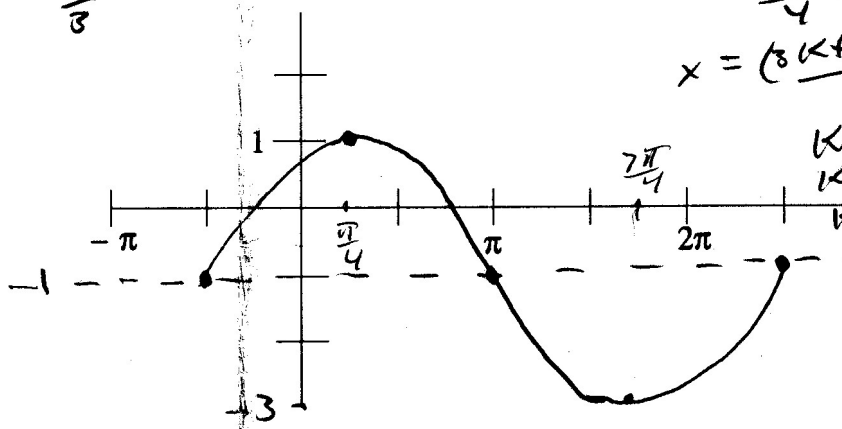
D_A $x \in \mathbb{R}$

R_A $-3 \le A(x) \le 1$

Per 3π

Amp 2

P.S. $\frac{\pi}{4}$ Right



B) $B(x) = -\sin\left(\frac{3}{2}x + \frac{\pi}{2}\right)$

$\frac{3}{2}\left(x + \frac{\pi}{3}\right) \overset{\text{Set}}{=} k\pi$

$x + \frac{\pi}{3} = \frac{2k\pi}{3}$

$x = \frac{(2k-1)}{3}\pi$

$k=0 \quad x = -\frac{\pi}{3}$

$k=1 \quad x = \frac{\pi}{3}$

$k=2 \quad x = \pi$

$\left(\frac{\pi}{3} + \pi = \frac{4\pi}{3}\right)\frac{1}{2}$

$\frac{2\pi}{3}$

$\frac{2\pi}{\frac{3}{2}} = \frac{4\pi}{3}$

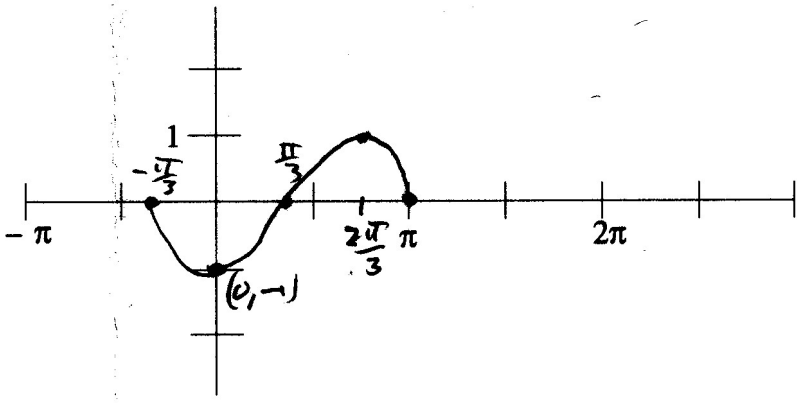
D_B $x \in \mathbb{R}$

R_B $-1 \le B(x) \le 1$

Per $\frac{4\pi}{3}$

Amp -1 (or 1)

P.S. $\frac{\pi}{3}$ Left



$K=0 \quad x=\frac{2\pi}{3}$

$K=1 \quad x=\frac{5\pi}{3}$

$K=2 \quad x=\frac{8\pi}{3}$

$K=-1 \quad x=-\frac{\pi}{3}$

$x-\frac{2\pi}{3} \overset{set}{=} k\pi$

$x=\frac{(3k+2)\pi}{3}$

C) $C(x) = \csc\left(x - \frac{2\pi}{3}\right) - 1$

D_C $x \neq \frac{(3k+2)\pi}{3}$

R_C $C(x) \le -2$ or $C(x) \ge 0$

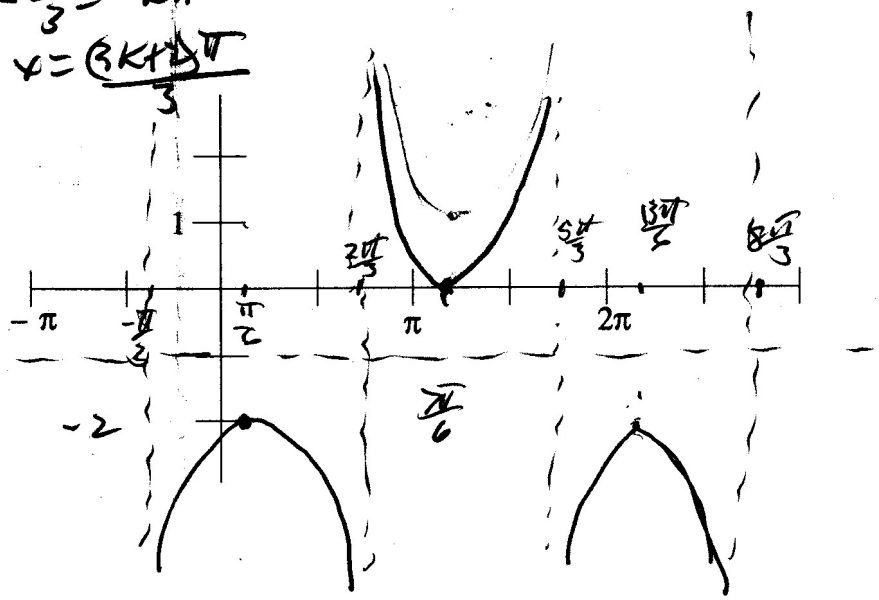
Per 2π

Amp 1

P.S. $\frac{2\pi}{3}$ Right

not cross y axis

-3



$\frac{4}{3}x = \frac{(2k+1)\pi}{2}$

$x = \frac{3}{8}(2k+1)\pi$

$\frac{(6k+3)\pi}{8}$

$K=-1$

$x=-\frac{3\pi}{8}$

$K=0$

$x=\frac{3\pi}{8}$

D) $D(x) = \tan\frac{4}{3}x$

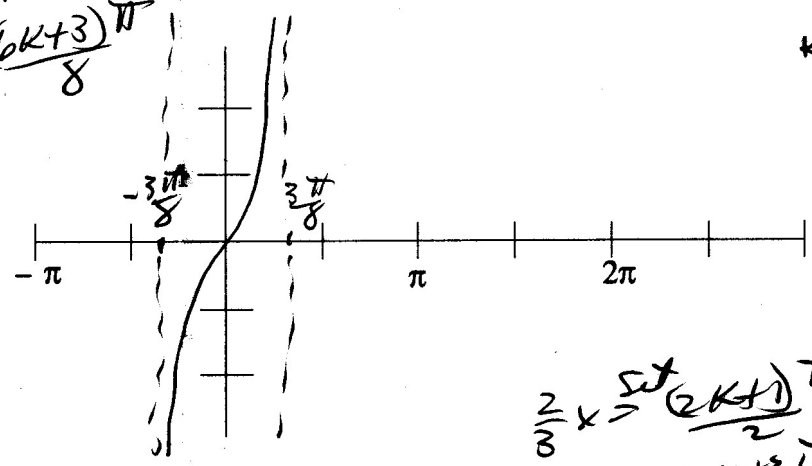
D_D $x \neq \left\{\frac{(6k+3)\pi}{8}\right\}$

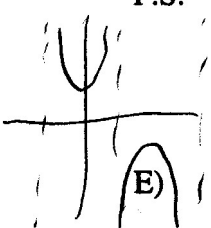
R_D $D(x) \in \mathbb{R}$

Per $\frac{3\pi}{4}$

Amp 1

P.S. zero





$\frac{2}{3}x \overset{set}{=} \frac{(2k+1)\pi}{2}$

$x = \frac{6k+3\pi}{4}$

$K=-1$

$x=-\frac{3\pi}{4}$

$K=0$

$x=\frac{3\pi}{4}$

$K=1 \quad \frac{9\pi}{4}$

E) $E(x) = \sec\frac{2}{3}x$

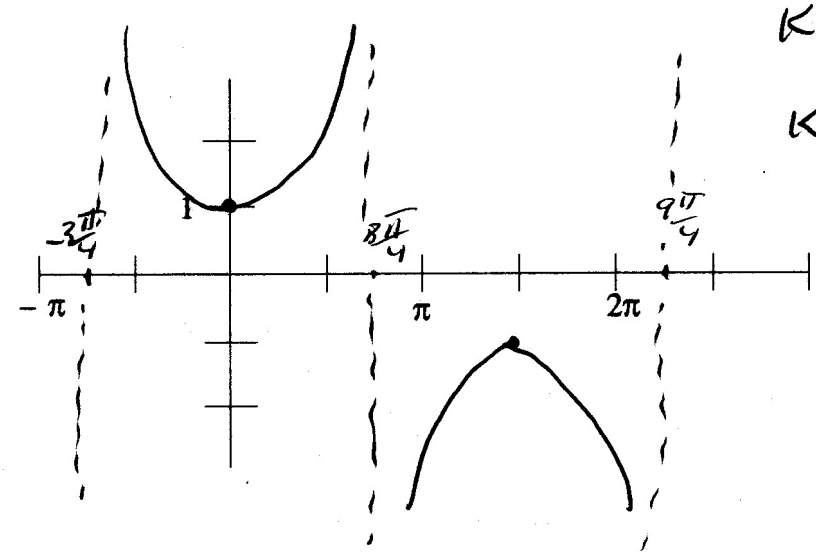
D_E $x \neq \left\{\frac{(6k+3)\pi}{4}\right\}$

R_E $E(x) \le -1$ or $E(x) \ge 1$

Per 3π

Amp 1

P.S. None



$\left(\frac{12\pi}{4}\right)\frac{1}{2}$

$\frac{3\pi}{2}$

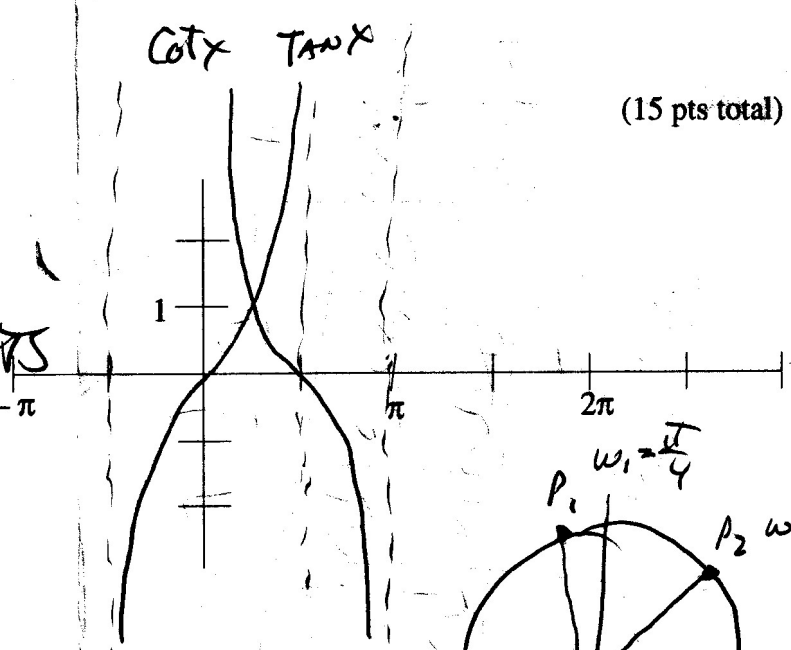
set wrong item

-3

II Draw a sketch of tan x and cot x on the supplied axis. (15 pts total)

Describe what must be done to tan x to make it coincide with cot x.

use appropriate Add/Subtract formula(s) to ... ***Prove*** that your changes would have the desired effect. (10 pts)

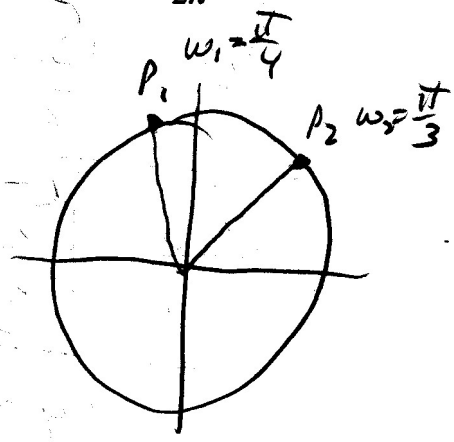


make Tan x negative and shift it right $\frac{\pi}{2}$

$$-\tan\left(x-\frac{\pi}{2}\right) = \frac{-\sin\left(x-\frac{\pi}{2}\right)}{\cos\left(x-\frac{\pi}{2}\right)}$$

$$= \frac{-\left(\sin x\cos\frac{\pi}{2} - \sin\frac{\pi}{2}\cos x\right)}{\cos x\cos\frac{\pi}{2} + \sin x\sin\frac{\pi}{2}}$$

$$= \frac{-(0 - \cos x)}{(0 + \sin x)} = \frac{\cos x}{\sin x} = \cot x.$$



III Given P_1 starts at $\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$ with $\omega_1 = \frac{\pi}{4}$ and P_2 starts at $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ with $\omega_2 = \frac{\pi}{3}$

$\frac{2\pi}{3} - \frac{\pi}{4} = \frac{(8-3)\pi}{12}$

(10 pts tot)

A) Find t when the two points first concide.

$\frac{\pi}{4}t + \frac{5\pi}{12} = \frac{\pi}{3}t \quad t = 5$

$\frac{5\pi}{12} = \frac{\pi}{12}t$

B) Confirm that P_1 and P_2 do, in fact, have the same coordinates at the t found in part A.

$\left(\cos\left(\frac{\pi}{4}\cdot 5 + \frac{2\pi}{3}\right), \sin(\ )\right) \overset{?}{=} \cos\left(\frac{\pi}{3}\cdot 5 + \frac{\pi}{4}\right), \sin(\ )$

$\cos\frac{5\pi}{4} + \frac{2\pi}{3} \qquad = \cos\left(\frac{5\pi}{3} + \frac{\pi}{4}\right),$

$\cos\frac{(15+8)\pi}{12} \qquad = \cos\frac{(20+3)\pi}{12}$

$\cos\frac{23\pi}{12}, \sin\frac{23\pi}{12} \qquad = \left(\cos\frac{23\pi}{12}, \sin\frac{23\pi}{12}\right)$ Same, Point.

EXTRA CREDIT ---------- **5 pts** ----------

As you know, to prove a function has period "a" you must show that $f(x+a) = f(x)$.

Given $f(x) = \cos 2x \sin x + \sin 2x \cos x$. Prove $f(x)$ has period $\frac{2\pi}{3}$.

$f(x) = \sin 2x\cos x + \sin x\cos 2x$

$= \sin 3x$ 2pts.

Now $f\left(x + \frac{2\pi}{3}\right) = \sin 3\left(x + \frac{2\pi}{3}\right)$

$= \sin(3x + 2\pi)$

$= \sin 3x = f(x)$ $\therefore$ f has per $\frac{2\pi}{3}$

typoon I d.

	2	3	T
100↑	1	0	1
90+	9	8	17
80	5	10	15
70	2	5	7
60	5	3	8
59	3	2	5

16 1

16

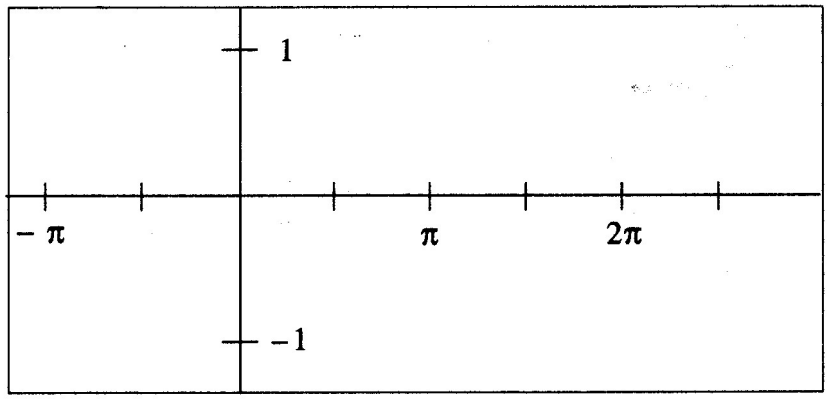
$\frac{1995}{25}$ $\frac{2228}{27}$

79.8 82.5 81.2

Just another
excellent exam

HONORS TRIGONOMETRY CHAPTER 3		EXAM CHAPTER 2 #1 R/C Pgs 32 - 35 #2 Do Part I on back of sked 10 / 9	#3 Do part II on back of sked 10/10	# 4,# 5 Pg 82-83 Change text instructions to read: **graph each and also state Domain & Range** 1 – 16 10/11
# 6 Same instructions as HW # 4, 5 Pg 92 1 - 12 Pg 88 1 – 6 **Generic graphs MUST be included!!!** 10/14	Pg 88 7 – 12 Pg 94 6, 5, 4, 3, 11 # 7 R/C Pg 95 - 98 # 8 10/15	Pg 98 - 100 1 - 10 # 9 **IMPORTANT LECTURE** 10/16	# 10, # 11 Pg 98 - 100 11 - 18 On problem 11, use 2 seconds and – 2 seconds instead of the + 3 and – 3 as listed in the text. 10/17	#12 ,13, 14, 15, 16 Jake's Graphing Adventure 10/18
Continue on Jakes's Graphing Adventure **Be prepared to turn in tomorrow.** 10/21	Begin **TEACUPS** # 17, 18, 19, 20, 21 10/22	Continue on TEACUPS 10/23	Continue on TEACUPS El Quizzo ??? 10/24	Begin: WEEKEND (not so) QUICKIE # 22, 23, 24, 25, 26 10/25
Turn in TEACUPSMore time on Weekend Quickie 10/28	Last day on WQ **You didn't do trig like this last year !** 10/29	Turn in the Quickie (end of period) Pg 104 - 105 1 - 6 # 27 10/30	Review 10/31	EXAM CHAPTER 3 2002 11/1

I For each of the following problems, draw and label an axis about 2 inches high and 4 inches long like this:



Plot the coordinates. Draw a smooth curve through the points. No straight lines and no sharp corners are allowed!

1)	$(-\pi, 0)$	2)	$(-\pi, -1)$	3)	$(-\pi, 0)$
	$(-\frac{\pi}{2}, -1)$		$(-\frac{\pi}{2}, 0)$		$(-\frac{\pi}{2}, -\frac{1}{2})$
	$(0, 0)$		$(0, 1)$		$(0, 0)$
	$(\frac{\pi}{2}, 1)$		$(\frac{\pi}{2}, 0)$		$(\frac{\pi}{2}, \frac{1}{2})$
	$(\pi, 0)$		$(\pi, -1)$		$(\pi, 0)$
	$(\frac{3\pi}{2}, -1)$		$(\frac{3\pi}{2}, 0)$		$(\frac{3\pi}{2}, -\frac{1}{2})$
	$(2\pi, 0)$		$(2\pi, 1)$		$(2\pi, 0)$

II A) Draw an axis labeled along the x-axis from $-\pi$ to 3π and along the y-axis from -2 to $+2$. Use the following domain values to create a list of coordinates for each of the functions. Plot the points for all functions on the one graph. Use a different color for each function.

$$x \;\varepsilon\; \left\{-\pi, -\tfrac{\pi}{2}, 0, \tfrac{\pi}{2}, \pi, \tfrac{3\pi}{2}, 2\pi, \tfrac{5\pi}{2}, 3\pi\right\}$$

$$f(x) = \cos x\,; \quad g(x) = 2\cos x\,; \quad h(x) = \frac{1}{2}\cos x$$

B) Draw an axis labeled along the x-axis from $-\pi$ to 3π and along the y-axis from -2 to $+2$. Use the following domain values to create a list of coordinates for each of the functions. Plot the points for all functions on the one graph. Use a different color for each function. (A calculator may be helpful. Decimal values may be used.

$$x \;\varepsilon\; \left\{-\pi, -\tfrac{2\pi}{3}, -\tfrac{\pi}{2}, -\tfrac{\pi}{3}, -\tfrac{\pi}{4}, 0, \tfrac{\pi}{4}, \tfrac{\pi}{3}, \tfrac{\pi}{2}, \tfrac{2\pi}{3}, \pi, \tfrac{5\pi}{4}, \tfrac{4\pi}{3}, \tfrac{3\pi}{2}, \tfrac{7\pi}{3}, 2\pi, \tfrac{5\pi}{2}, 3\pi\right\}$$

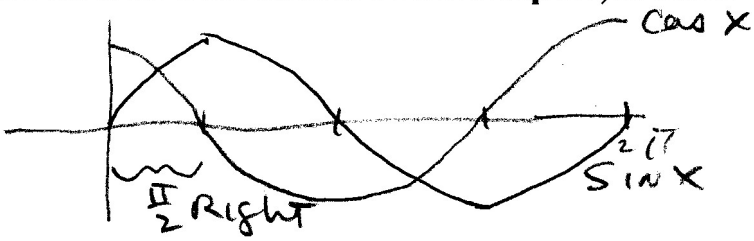
$$f(x) = \cos x\,; \quad g(x) = \cos 2x\,; \quad h(x) = \cos 3x$$

C) Draw an axis labeled along the x-axis from $-\pi$ to 3π and along the y-axis from -2 to $+2$. Use the following domain values to create a list of coordinates for each of the functions. Plot the points for all functions on the one graph. Use a different color for each function. (A calculator may be helpful. Decimal values may be used.

$$x \;\varepsilon\; \left\{-\pi, -\tfrac{2\pi}{3}, -\tfrac{\pi}{2}, -\tfrac{\pi}{3}, -\tfrac{\pi}{4}, 0, \tfrac{\pi}{4}, \tfrac{\pi}{3}, \tfrac{\pi}{2}, \tfrac{2\pi}{3}, \pi, \tfrac{5\pi}{4}, \tfrac{4\pi}{3}, \tfrac{3\pi}{2}, \tfrac{7\pi}{3}, 2\pi, \tfrac{5\pi}{2}, 3\pi\right\}$$

$$f(x) = \sin x - 1\,; \quad g(x) = \sin x\,; \quad h(x) = \sin x + 1$$

Do the work for these sections on the newsprint, as usual.



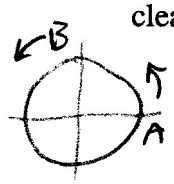
$\cos(x - \frac{\pi}{2}) =$
$\cos x \cos\frac{\pi}{2} + \sin x \sin\frac{\pi}{2}$
$0 + \sin x$
$\sin x$.

4 pts prob

II Some people say that the cosine function is just a sine function which is out of phase. Sketch both functions on the same axis and clearly identify each. Determine the phase shift needed to illustrate their point and PROVE that you are correct. (10 pts)

III At time $t = 0$, point A is at $(1, 0)$ and point B is at $(0, 1)$. $\omega_A = \frac{2\pi}{3}$ and $\omega_B = \frac{\pi}{2}$.

Determinethe coordinates of the **second** meeting. (In order to earn full credit, you must be very clear in your steps toward solution. Draw pictures, label things, describe what your are doing, etc. (10 pts)



A is faster will catch B after making up $\frac{\pi}{2}$ distance.
The we need 1 full Lap.
dist A = dist B + $\frac{5\pi}{2}$

$\frac{2\pi}{3}t = \frac{\pi}{2}t + \frac{5\pi}{2}$ → $t = 15$ sec.

$\frac{4}{6}t - \frac{3}{6}t = \frac{5}{2}$

$\cos(5 \cdot \frac{2\pi}{3})$, $\sin(5 \cdot \frac{2\pi}{3})$

$\cos 10\pi$, $\sin 10\pi$

$(1, 0)$ is where they meet the 2nd time.

IV Define, compare, and contrast the terms: (5 pts)

Linear Velocity
Angular Velocity
Rotational Velocity

$\omega = \frac{2\pi}{T} =$ Same
where T is time for 1 Rev
& ω is linear velocity

EXTRA CREDIT ■■■■■■■■■■■■■■■■ 5 pts ■■■■■■■■■■■■■■■■

Given: $F(x) = \sin 4x + \cos 3x$

Determine the fundamental period of $F(x)$ then **PROVE** your are correct.

fundamental period is LCM of $\frac{\pi}{2}$ and $\frac{2\pi}{3}$ which is 2π

$4 \cdot \frac{\pi}{2} = 2\pi = 3 \cdot \frac{2\pi}{3}$

$f(x + 2\pi) = \sin 4(x + 2\pi) + \cos 3(x + 2\pi)$
$= \sin(4x + 8\pi) + \cos(3x + 6\pi)$
$= \sin 4x \cos 8\pi + \sin 8\pi \cos 4x + \cos 3x \cos 6\pi - \sin 3x \sin 6\pi$
$= \sin 4x + 0 + \cos 3x - 0$

$f(x + 2\pi) = f(x)$

	2	5	T	
100	0	0	0	98
90	8	6	14	
80	4	8	12	
70	7	7	14	
60	2	5	7	
59	1	2	3	
	81.8	77.6	79.5	44

$\frac{1800}{22}$ $\frac{2174}{28}$

I thought this was extremely easy exam – barely acceptable in 2nd per. obviously not in 5th. may be add 2 more graphing, R & D etc Quizzes last 2 days before exam. many last pts there for slappeyness D_c $C(x) \in \mathbb{R}$ type of stuff.

II c $22\cdot\frac{\pi}{3}\cdot 3 + 3\cdot 22\cdot\frac{\pi}{4}$

$\frac{11}{\cancel{66}}\pi\left(\frac{4+3}{\cancel{12}}\right)$ $\cancel{12}$

$\frac{77\pi}{2}$

Do the work for the remainder of the exam on the newsprint as usual.

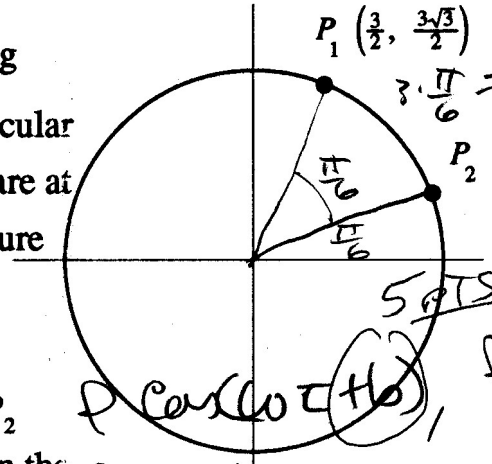
a) $dP_1 = dP_2 + \frac{11\pi}{2}$

$3\cdot\frac{\pi}{3}\tau = 3\cdot\frac{\pi}{4}\tau + \frac{11\pi}{2}$

$\frac{1}{4}\tau = \frac{11}{2}$ $\tau = 22$

II

Given the points P_1 and P_2 rotating around the circle with uniform circular motion. At time $t = 0$, P_1 and P_2 are at the locations indicated on the picture on the right.



(25 pts tot)

5 pts

$P\cos(\omega\tau + b)$, $P\sin(\omega\tau + b)$

10 a) When (find t) will P_1 and P_2 occupy the same location on the circle?

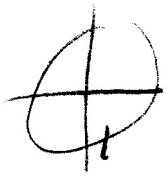
$\left(3\cos\left(\frac{\cancel{22}\,11}{\cancel{4}\,2}\cdot\frac{\pi}{3} + \frac{\pi}{6}\right), 3\sin\left(22\cdot\frac{\pi}{4} + \frac{\pi}{6}\right)\right)$

10 b) What are the coordinates of that point?

$3\cos\frac{34\pi}{6}$, $3\sin\left(\frac{17\pi}{3}\right)$

5 c) What was the total distance covered by points P_1 ~~and P_2~~?

$\left(3\cos\left(\frac{5\pi}{3} + \frac{12\pi}{3}\right), 3\sin\frac{5\pi}{3}\right)$



together?

$\left(\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right)$

Extra Credit ..... 5 pts .....

$\frac{\pi}{3}\left(x + \frac{\pi}{3}\right)$

Given: $f(x) = a\cos(bx + c)$ and $g(x) = d\cos(ex + f)$

$\frac{\pi}{4}\left(x + \frac{\pi}{6}\right)$

Where $f(x)$ describes the movement of P_1 in section II above and $g(x)$ describes the movement of P_2 in **section II** above.

$a = 3$, $d = 3$ } Radius of circle.

Find the explicit values of the constants: a, b, c, d, e, and f.

Explain, of course.

$b = \frac{\pi}{3}$ ω P_1

$e = \frac{\pi}{4}$ ω P_2

$c = \frac{\pi^2}{9}$

$f = \frac{\pi^2}{24}$

	R	S	T	
100	0	0	0	99
90	8	13	21	
80	6	5	11	
70	3	5	8	
60	2	0	2	
59	3	0	3	36

1793	2001	3794
22	23	45
81.5	87	84.3

Great exam!

I gave both quizes
1 day later than scheduled.
This gave the students an opportunity to
work unimpeded on both major
ditto...

Do the work for the remainder of the exam on the newsprint as usual.

Cosine is at 1 $\frac{\pi}{2}$ later than sine. $\cos(x - \frac{\pi}{2}) = \sin x$

$\cos x \cos\frac{\pi}{2} + \sin x \sin\frac{\pi}{2} =$

$0 + \sin x$

$\sin x = \sin x$

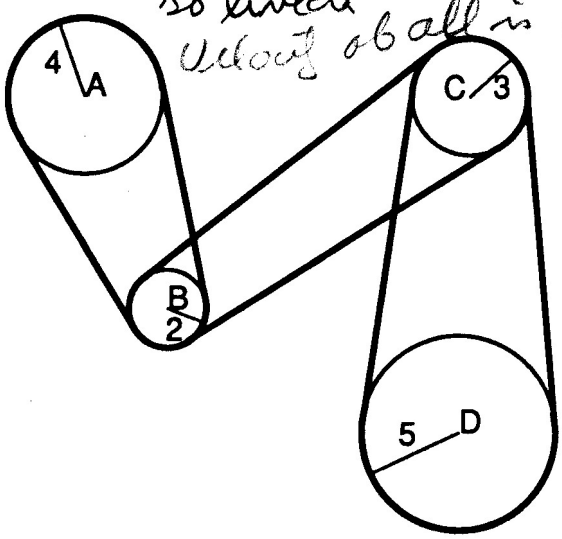
II Some people say "A cosine curve is nothing more than a sine curve which is out of phase." Explain what they mean and **prove** that they are correct. (15 pts)

III Given the four connected pulleys as shown. Pulley "A" is "driving" at three revolutions per four seconds in a counter-clockwise direction. Pulleys "B" and "C" each have two slots to accomodate both of their belts. Determine the rotational velocities of all four pulleys. (10 pts)

3 Rev = 4 sec

1 Rev = $\frac{4}{3}$ sec $\therefore \omega_A = \frac{2\pi}{\frac{4}{3}} = \frac{3\pi}{2}$

so linear velocity of all is 6π



$\rho_x \omega_x = 6\pi$

$\omega_A = \frac{3\pi}{2}$

$\omega_B = 3\pi$

$\omega_C = 2\pi$

$\omega_D = \frac{6\pi}{5}$

Extra Credit ..... 5 pts .....

On my clock, the hour hand moves once ever hour, the minute hand moves once every minute, and the second hand moves once every seven seconds. If they all move to 12 at exactly midnight on Sunday, on what day and hour will all three next be pointing to 12?

Hour hand Points at 12 every 12 hrs = 720 min

minute hand " " = 60 min

second hand " " = 7 min

we want the smallest # 720, 60, 7 divide into. ie Least Common multiple

$720 = 72 \cdot 10$

$36 \cdot 2^2 \cdot 5$

$2^2 \cdot 3^2 \cdot 2^2 \cdot 5$

$2^4 \cdot 3^2 \cdot 5$

$60 = 4 \cdot 3 \cdot 5$

$2^2 \cdot 3 \cdot 5$

$4 \cdot 3 \cdot 7$

$\frac{2^{\cancel{4}\,2} \cdot 3^{\cancel{2}} \cdot \cancel{5} \cdot 7 \text{ min}}{60 \text{ min } \cancel{2^2} \cdot \cancel{3} \cdot \cancel{5}} \; \frac{1 \text{ hr}}{}$

84 hrs

$24 \overline{)84}$ = 3, 72, 12

$3\frac{1}{2}$ days

Thursday at Noon.

	2	5	T
100	0	1	1
90	7	11	18
80	8	4	12
70	4	4	8
60	3	1	4
59	1	1	2

100

55

$\frac{1847}{23}$ $\frac{1901}{22}$ $\frac{3748}{45}$

83.3

80.3 $\frac{86.41}{\text{Ans}}$

Honors Trigonometry Name Trig ch 3

Convince *me* that **you** understand the concept. Per _____ Date 10/31/96

CHAPTER #3

I This section covers all of the first two pages of this exam. For *each* problem, supply **domain, range, amplitude, period** and **phase shift (including direction)** and **draw** the graph *in the areas provided.* Be sure your drawing crosses or touches the Y-axis. You may do the work for this exam *directly on this* exam paper rather than on the "newsprint" as usual. If you do work on newsprint be sure it is labeled with its problem number. **Indicate significant coordinates You *may* used decimals for you coordinate responses.** Each problem in this section is worth 15 points.

A) $A(x) = 2\sin\left(\frac{3}{4}x - \frac{3\pi}{16}\right) + 1$

$\frac{2\pi}{\frac{3}{4}} = \frac{8\pi}{3}$

must cross 2pts

$\frac{3}{4}\left(x - \frac{\pi}{4}\right) = \frac{k\pi}{2}$

$x - \frac{\pi}{4} = \frac{4k\pi}{3}$

$x = \frac{16k\pi + 3\pi}{12}$

$\frac{(16k+3)\pi}{12}$

$k=0$ $\frac{\pi}{4}$

$k=1$ $\frac{19\pi}{12}$

$k=2$

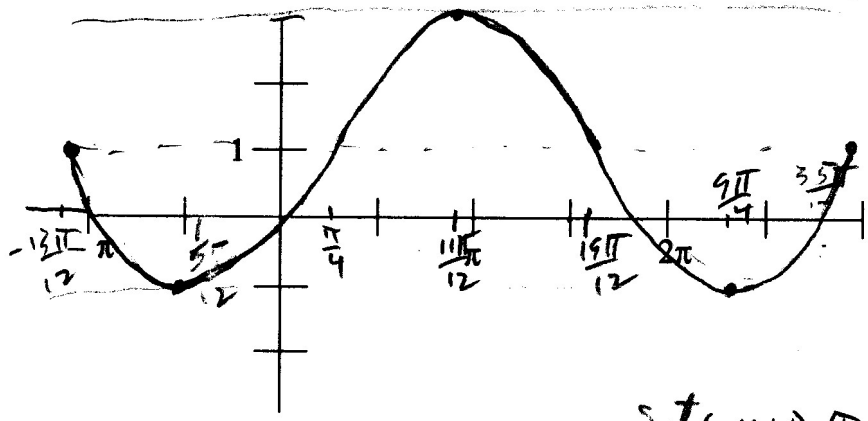
D_A $x \in \mathbb{R}$ $2\sin\frac{3}{4}\left(x - \frac{\pi}{4}\right) + 1$

R_A $-1 \le A(x) \le 3$

Per $\frac{8\pi}{3}$

Amp 2

P.S. $\frac{\pi}{4}$ Right



$\frac{2}{3}\left(x + \frac{\pi}{3}\right) = \frac{(2k+1)\pi}{2}$

$\frac{2\pi}{\frac{2}{3}} = 3\pi$

$k=-1$ $-\frac{13\pi}{12}$ $x + \frac{\pi}{3} = \frac{6k\pi}{4} + \frac{3\pi}{4}$

$k=0$ $\frac{5\pi}{12}$ $\frac{18k\pi + 5\pi}{12}$

$k=1$ $\frac{23\pi}{12}$ $\frac{(18k+5)\pi}{12}$

B) $B(x) = -\cos\left(\frac{2}{3}x + \frac{2\pi}{9}\right)$

$\frac{2}{3}\left(x + \frac{\pi}{3}\right)$

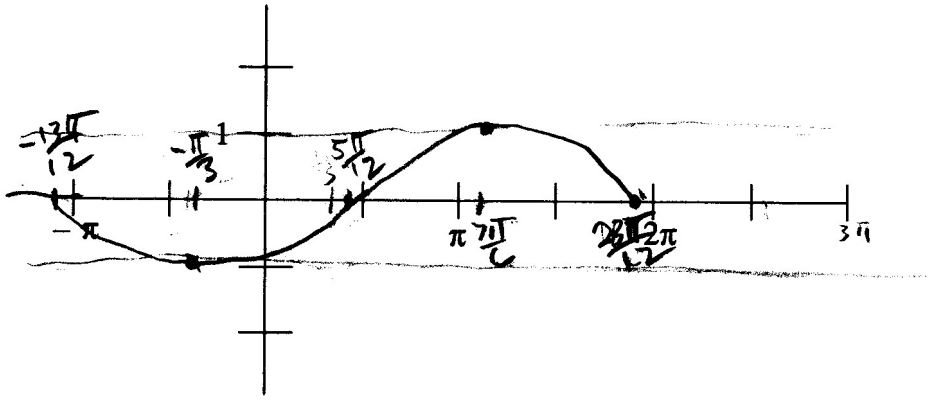
D_B $x \in \mathbb{R}$

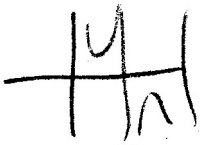
R_B $-1 \le B(x) \le 1$

Per 3π

Amp 1 (or −1)

P.S. $\frac{\pi}{3}$ Left





C) $C(x) = \csc\left(2x + \frac{\pi}{4}\right) + 1$

$2\left(x+\frac{\pi}{8}\right) \neq k\pi$

$x + \frac{\pi}{8} = \frac{4k\pi}{8}$

$x = \frac{(4k-1)\pi}{8}$

$k=0 \quad -\frac{\pi}{8}$

$k=1 \quad \frac{3\pi}{8}$

$k=2 \quad \frac{7\pi}{8}$

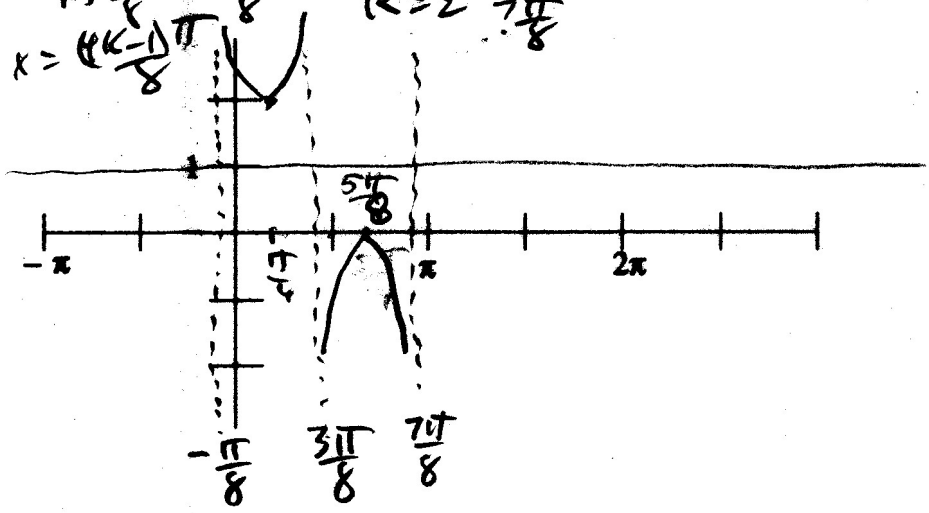
D_C $x \neq \frac{(4k-1)\pi}{8}$ $2\left(x+\frac{\pi}{8}\right)$

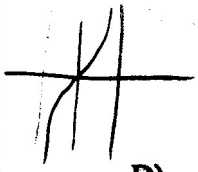
R_C $C(x) \leq 0$ or $C(x) \geq 2$

Per π

Amp 1

P.S. $\frac{\pi}{8}$ Left





D) $D(x) = \tan\frac{4}{3}x$

$\frac{4}{3}x \neq \frac{(2k+1)\pi}{2}$

$x = \frac{(6k+3)\pi}{8}$

$k=-1 \quad -\frac{3\pi}{8}$

$k=0 \quad \frac{3\pi}{8}$

$\frac{\pi}{\frac{4}{3}} = \frac{3\pi}{4}$

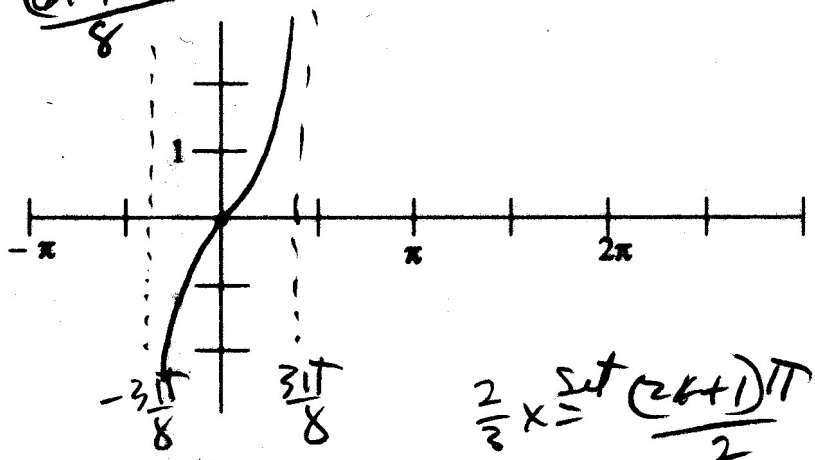
D_D $x \neq \frac{(6k+3)\pi}{8}$

R_D $D(x) \in \mathbb{R}$

Per $\frac{3\pi}{4}$

Amp 1

P.S. None



E) $E(x) = \sec\frac{2}{3}x$

$\frac{2}{3}x \neq \frac{(2k+1)\pi}{2}$

$x = \frac{(6k+3)\pi}{4}$

$k=-1 \quad -\frac{3\pi}{4}$

$k=0$

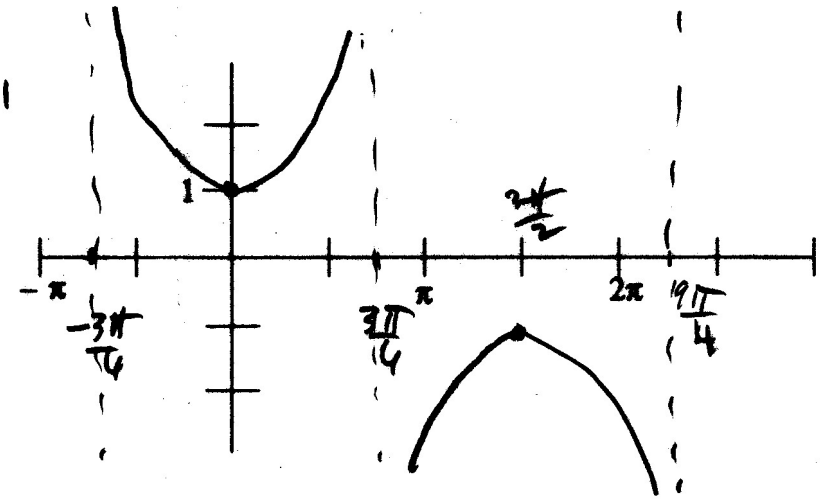
D_E $x \neq \frac{(6k+3)\pi}{4}$

R_E $E(x) \leq -1$ or $E(x) \geq 1$

Per $\frac{2\pi}{\frac{2}{3}} = 3\pi$

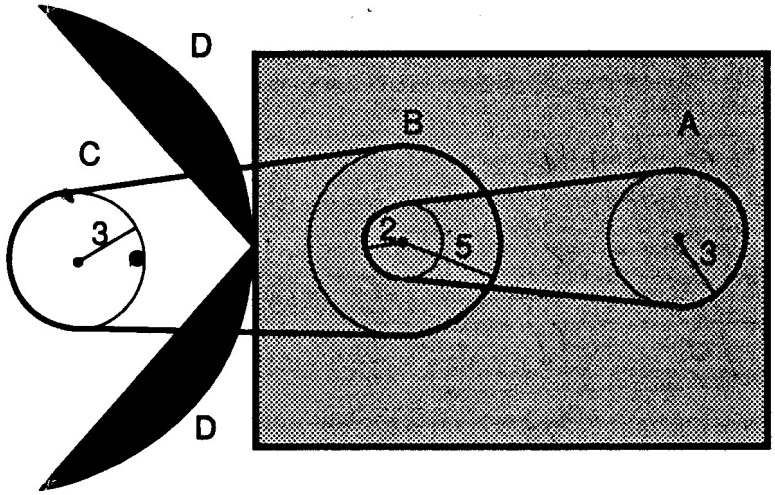
Amp 1

P.S. None



II **(25 pts total)**

The figure on the right is a rotating emergency light. The dot on wheel C is the light source. The black objects labeled D are mirrors which reflect the bright light this direction from the dot. Wheel A is the motor wheel and it rotates at a constant speed of 15 RPM. Wheel B alters the speed of the rotating light slightly because it is connected to the light wheel by its 5 inch pulley and to the drive wheel by its 2 inch radius pulley. The drive wheel A and the rotating light wheel C each have a radius of 3 inches.



When the motor is turned on, the red light is at the place indicated in the picture which is as close as it gets to the shaded housing, 5 inches away.

Your job is to answer the questions below. Include very brief justification for each section. Be sure *I* understand your logic. Your answers should be in seconds rather than minutes.

A) Determine the angular velocities on wheel A. Singular, in Seconds

B) Determine the linear velocities on wheel A.

C) Determine both linear velocities on wheel B. (Careful ... They are not the Same)

D) Determine the linear velocity on wheel C.

E) Determine the angluar velocity on wheel C.

F) Write the equation in terms of sine with a positive amplitude which describes the distance the dot on wheel C is from the shaded housing.

G) Draw that graph of your function on the newsprint. Indicate significant coordinates.

Extra Credit ..... **5 pts** .....

For certain domain values, the functions $f(x) = \sin x$ and $g(x) = \sin 2x$ from $x = 0$ to $x = 2\pi$ have the same range value. For example, $f(\pi)$ and $g(\pi)$ both have the range value of zero, What are the other domain and range combinations?

II 15 Rev = 60 sec.

1 Rev = 4 sec = 'T'

$\omega_A = \frac{2\pi}{4} = \frac{\pi}{2}$ (a)

(c) $3\omega_A = \frac{3\pi}{2}$ (b) $= 2\omega_B$ Same linear velocity

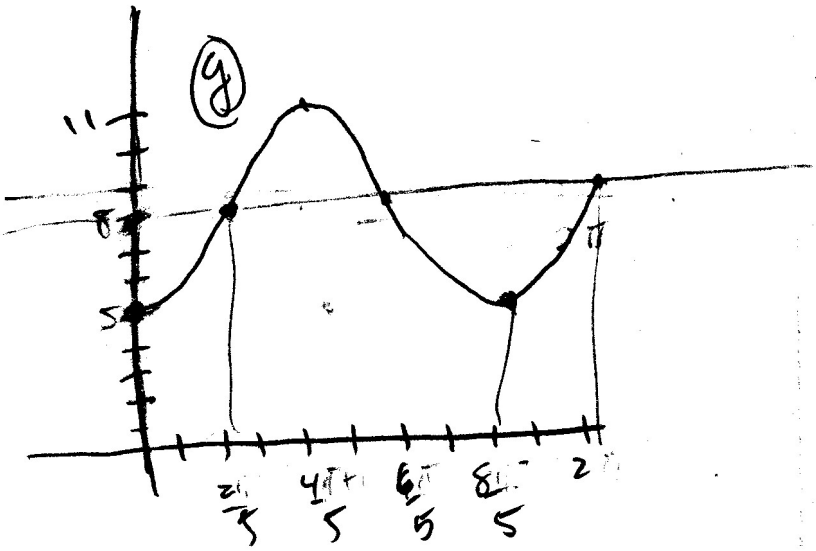
$\frac{3\pi}{2}$ and $\omega_B = \frac{3\pi}{4}$ the wheel can have only 1 ω

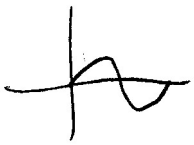
$5\omega_B = \frac{15\pi}{4}$ Linear velocity outside pully

(D) $3\omega_C = \frac{15\pi}{4}$ connected

(E) $\omega_C = \frac{5\pi}{4}$

$\frac{2\pi}{\frac{5\pi}{4}} = \frac{8}{5}$ = New period.





PS $\frac{2}{5}$ Right

axis up 8 Amp 3

$f(x) = 3\sin\frac{5\pi}{4}\left(x - \frac{2}{5}\right) + 8$

(f) $3\sin\left(\frac{5\pi}{4}x - \frac{\pi}{2}\right) + 8$

notice that $\frac{\pi}{2}$ due to starting point of the Dot.

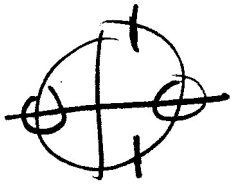
Extra.

$\sin x = \sin 2x$

$\sin x - 2\sin x\cos x = 0$

$\sin x(1 - 2\cos x) = 0$

$\sin x = 0$ or $\cos x = \frac{1}{2}$



$x = 0$ Trivial case

$x = \frac{\pi}{3}$

$x = \pi$ Given

$x = \frac{5\pi}{3}$

100		98
90	14	
80	8	
70	4	
60	0	
59	5	12

$\frac{2478}{30}$ 82.6

Great extra!

changes on section II as shown on opposite side of this page.

94

	5	6	T
100	0	0	1
90	4	2	6
80	11	7	18
90	6	7	13
60	1	1	2
59		3	3

103

42

$\frac{1921}{\cancel{1818}}$ / 23 — 835

$\frac{1499}{\cancel{1597}}$ / 20 — ~~76.0~~ 74.7

$\frac{3415}{43} = 79.4$

No changes.

III is too easy but there is no need to change it. They need the easy points.

Generally I took 2 if 3 answers were given and 5 pts if less was there.

91

Excellent exam. I thought everyone would be in the 90's. They have trouble with the fractions.

Most had real trouble with VI. They didn't make the connection of the letters a, b, c, d and the picture.

When they see solutions, they will be upset w/ themselves.

	2	5	T
100	0	1	1
90	2	13	15
80	8	3	11
70	4	5	9
60	2	2	4
59	0	0	0

102

62

$\frac{1291}{16}$ $\frac{2076}{24}$ $\frac{3367}{40}$

80.7 86.5 84.2

98

	4	5	T
100	1	0	1
90	2	7	9
80	11	9	20
70	9	6	15
60	0	1	1
59	0	3	3

103

27

$\frac{1900}{23}$ $\frac{1960}{24}$ $\frac{3860}{47}$

82.6 81.7 82.1

a really easy exam.

Honors Trigonometry Name Trg ch3 98

Per _____ Date _____

Convince *me* that **you** understand the concept.

CHAPTER #3

Show x values of significant points

I This section covers all of the first two pages of this exam. For *each* problem, supply **domain, range, amplitude, period** and **phase shift (including direction)** and **draw** the graph *in the areas provided.* Be sure your drawing crosses or touches the Y-axis. You are to do the work for section I *directly on this* exam paper rather than on the "newsprint" as usual. ~~Do all work for each problem on newsprint, but put answers in the space provided.~~ Each problem in this section is worth 15 points.

A) $A(x) = 2\cos\left(\frac{3}{4}x - \frac{3\pi}{16}\right) + 1$

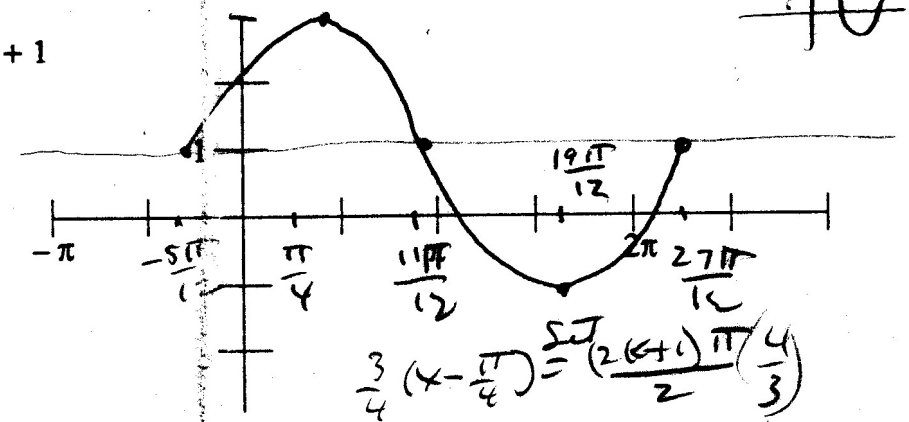
D_A $x \in \mathbb{R}$

R_A $-1 \le A(x) \le 3$

Per $\frac{8\pi}{3}$

Amp 2

P.S. $\frac{\pi}{4}$ Right



$\frac{3}{4}\left(x - \frac{\pi}{4}\right) \overset{\text{set}}{=} \frac{(2k+1)\pi}{2}\left(\frac{4}{3}\right)$

$x - \frac{\pi}{4} = \frac{4k\pi}{3} + \frac{2\pi}{3}$

$= \frac{16k\pi + 8\pi + 3\pi}{12}$

$x = \frac{(16k+11)\pi}{12}$

B) $B(x) = -\sin\left(\frac{2}{3}x + \frac{2\pi}{9}\right)$

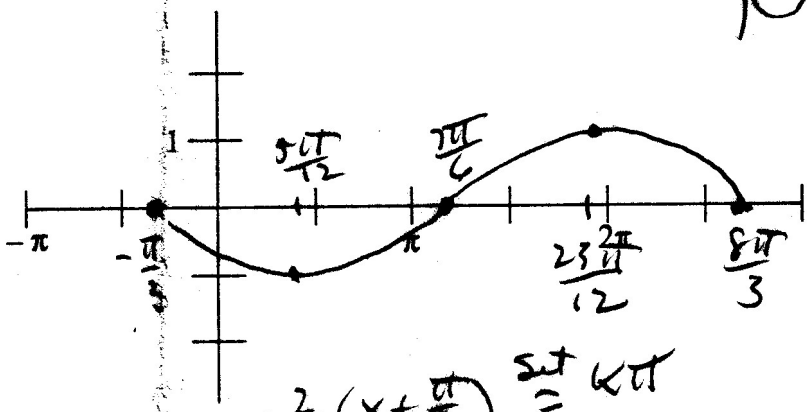
D_B $x \in \mathbb{R}$

R_B $-1 \le B(x) \le 1$

Per 3π

Amp 1

P.S. $\frac{\pi}{3}$ Left



$\frac{2}{3}\left(x + \frac{\pi}{3}\right) \overset{\text{set}}{=} k\pi$

$x + \frac{\pi}{3} = \frac{3}{2}k\pi$

$x = \frac{3k\pi}{2} - \frac{\pi}{3}$

$x = \frac{(3k-2)\pi}{6}$

C) $C(x) = \sec\left(2x + \frac{\pi}{4}\right) + 1$

D_C $x \notin \left\{\frac{(4K+1)\pi}{8}\right\}$

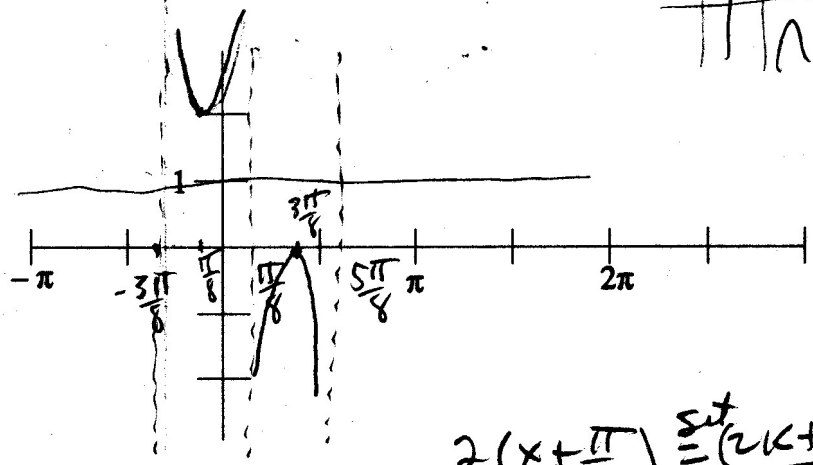
-2 written wrong.

R_C $C(x) \le 0$ or $C(x) \ge 2$

Per π

Amp 1

P.S. $\frac{\pi}{8}$ Left



$2\left(x + \frac{\pi}{8}\right) \overset{\text{set}}{=} \frac{(2K+1)\pi}{2}$

$x + \frac{\pi}{8} = \frac{2K\pi}{4} + \frac{\pi}{4}$

$x = \frac{4K\pi}{8} + \frac{\pi}{8}$

$\frac{(4K+1)\pi}{8}$

D) $D(x) = \cot\frac{4}{3}x$

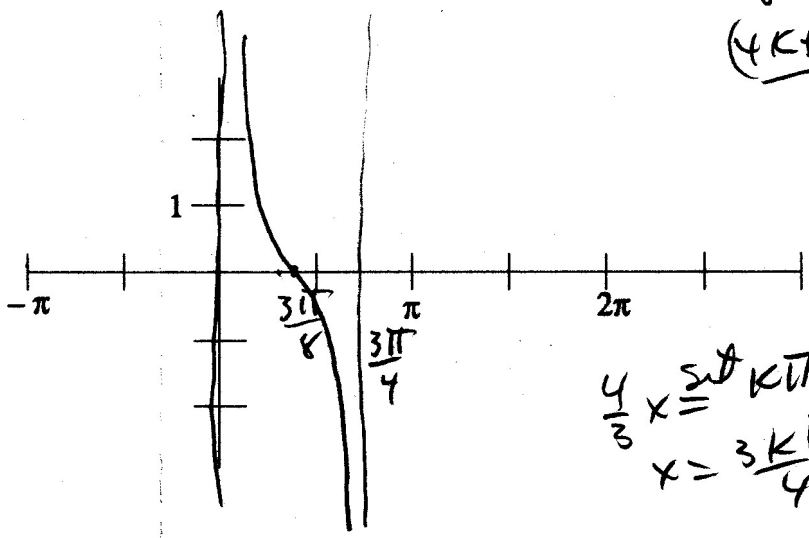
D_D $x \notin \left\{\frac{3K\pi}{4}\right\}$

R_D $D(x) \in \mathbb{R}$

Per $\frac{3\pi}{4}$

Amp 1

P.S. None



$\frac{4}{3}x \overset{\text{set}}{=} K\pi$

$x = \frac{3K\pi}{4}$

E) $E(x) = \csc\frac{2}{3}x$

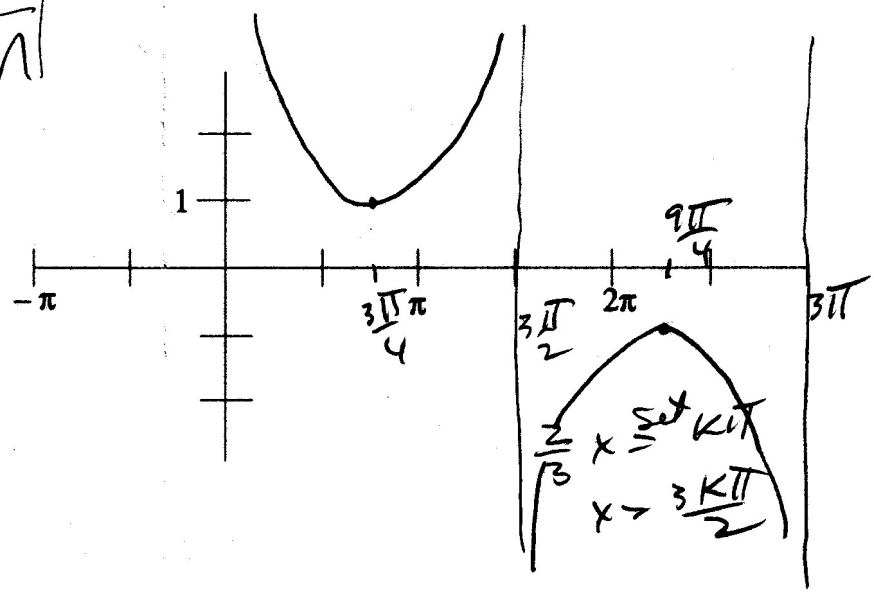
D_E $x \notin \left\{\frac{3K\pi}{2}\right\}$

R_E $|E(x)| \ge 1$

Per 3π

Amp 1

P.S. None



$\frac{2}{3}x \overset{\text{set}}{=} K\pi$

$x = \frac{3K\pi}{2}$

You may write on this portion of the exam. Per 3π d = −1

~~Do the work for these sections~~ on the newsprint, as usual.

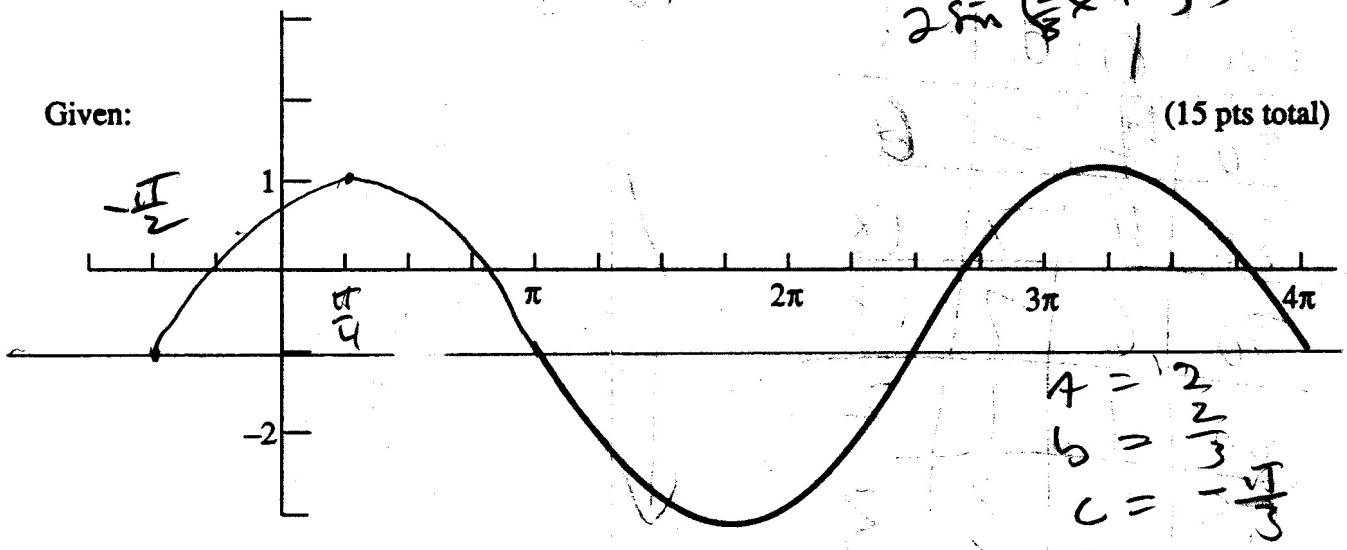
However, you may work 1 if you ~~[illegible]~~ prefer.

Amp = 2

$A(x) = 2\sin\frac{2}{3}(x+\frac{\pi}{2})-1$

$2\sin(\frac{2}{3}x+\frac{\pi}{3})-1$

II Given: (15 pts total)



B)
a = 2
$b = \frac{2}{3}$
$c = \frac{\pi}{6}$
d = −1

A = 2
$b = \frac{2}{3}$
$c = -\frac{\pi}{3}$
D = −1

A) Also given: $A(x) = a\sin(bx - c) + d$ and $a > 0$ and $-\pi < c < 0$
Find the values of a, b, c, and d. PS $\frac{\pi}{2}$ Left

−8 sum
b, c b, c

B) Also given: $B(x) = a\cos(bx - c) + d$ and $a > 0$ and $c > 0$
Find the values of a, b, c, and d. PS $\frac{\pi}{4}$ Right.

−3 sum
c, c

$B(x) = 2\cos\frac{2}{3}(x-\frac{\pi}{4})-1$

$2\cos(\frac{2}{3}x-\frac{\pi}{6})-1$

III

At time t = 0, point A is at (1 , 0) and point B is at (0 , 1). $\omega_A = \frac{2\pi}{3}$ and $\omega_B = \frac{\pi}{2}$.

+2π = −5

Determine the *coordinates* **of each of the first three** meetings. (In order to earn full credit, you must be very clear in your steps toward solution. Draw pictures, label things, describe what your are doing, etc.)

1st $d_A = d_B + \frac{\pi}{2}$ 2nd $d_A = d_B + \frac{\pi}{2} + 2\pi$ 3rd $d_A = d_B + \frac{\pi}{2} + 4\pi$

$\frac{1}{6}t = \frac{1}{2}$ $\frac{1}{6}t = \frac{5}{2}$ $\frac{1}{6}t = \frac{9\pi}{2}$ (10 pts)

$t = 3$ $t = 15$ $t = 27$

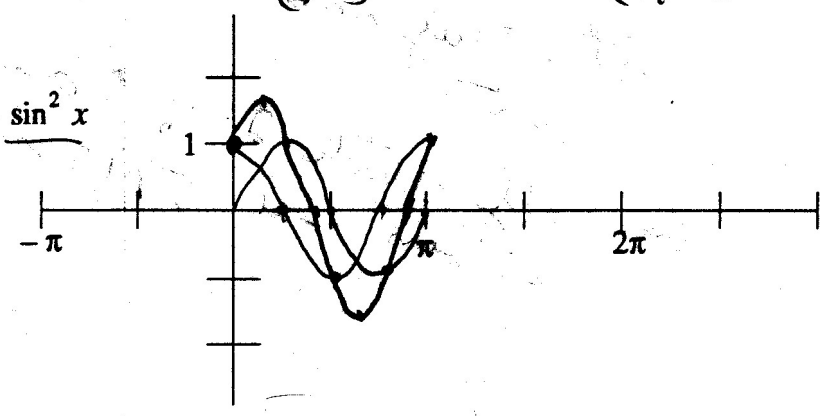
EXTRA CREDIT ---------- **5 pts** ----------

$(\cos 3\cdot\frac{2\pi}{3}, \sin 2\pi)$ $\cos 15\cdot\frac{2\pi}{3}, \sin 10\pi$ $\cos 27\cdot\frac{2\pi}{3}, \sin 18\pi$

(1, 0) (1, 0) (1, 0)

Sketch ***exactly*** 1 period of:

$$f(x) = \underbrace{\cos^2 x}_{\cos 2x} + \underbrace{2\sin x\cos x}_{\sin 2x} - \underbrace{\sin^2 x}$$



Cos 2x Both have
Sin 2x Per π

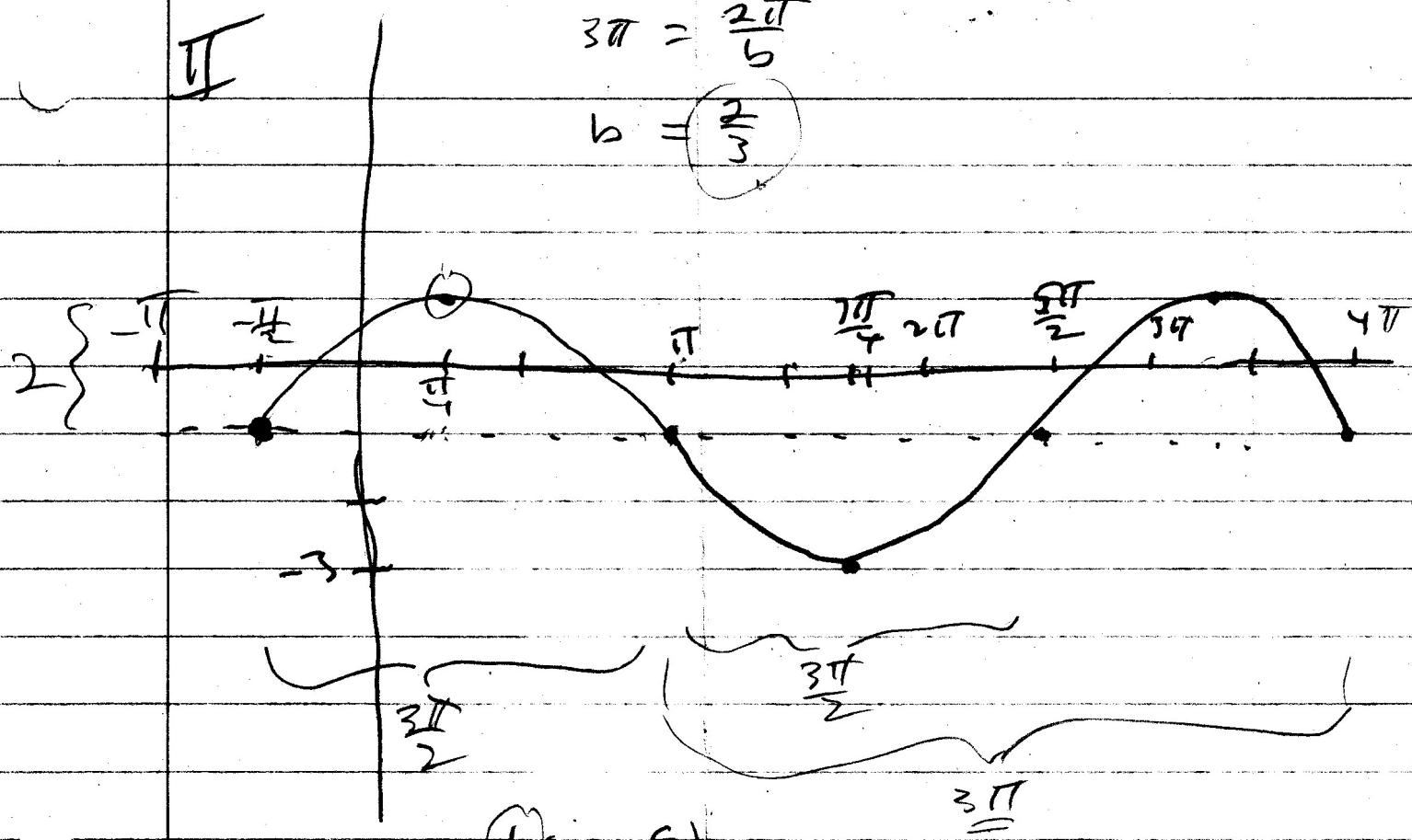
$$\frac{\text{new}}{\text{per}} = \frac{2\pi}{b}$$

water wheel

$$3\pi = \frac{2\pi}{b}$$

$$b = \frac{2}{3}$$

II



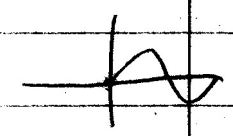
$$b\left(x - \frac{c}{b}\right)$$

$a = 2$

$d = 1$

$$A(x) = a\sin(bx - c) + d$$

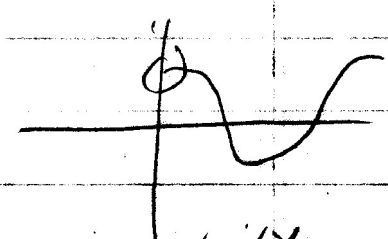
$$2\sin\frac{2}{3}\left(x + \frac{\pi}{2}\right) - 1$$



$$2\sin\left(\frac{2}{3}x + \frac{\pi}{3}\right) - 1$$

$$\underset{a}{2}\sin\left(\underset{b}{\frac{2}{3}}x - \underset{c}{\left(-\frac{\pi}{3}\right)}\right) \underset{d}{-1}$$

$$B(x) = a\cos(bx - c) + d$$



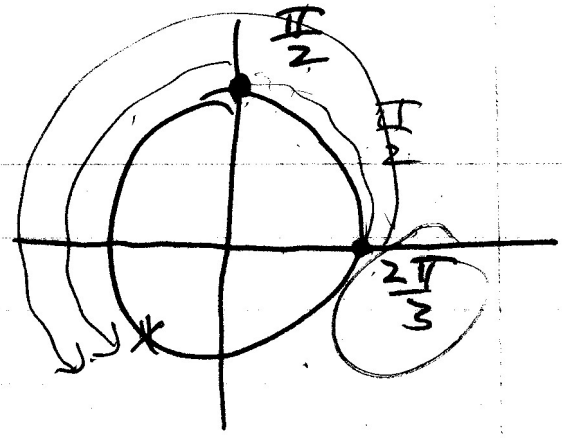
$$a\cos b\left(x - \frac{c}{b}\right) + d$$

$$2\cos\frac{2}{3}\left(x - \frac{\pi}{6}\right) - 1$$

Phase shift

$$2\cos\left(\frac{2}{3}x - \frac{\pi}{6}\right) - 1$$

$$\underset{a}{2}\quad \underset{b}{\frac{2}{3}}\quad \underset{c}{\frac{\pi}{6}}\quad \underset{d}{-1}$$



$(\cos \omega t, \sin \omega t)$

$\cos \frac{2\pi}{3} \cdot 3, \sin \frac{2\pi}{3} \cdot 3)$

$\cos 2\pi, \sin 2\pi$

$(1, 0)$

$RT = d$

$$\frac{2\pi}{3} t = \frac{\pi}{2} t + \frac{\pi}{2}$$

$$\frac{4\pi}{6} t = \frac{3\pi}{6} t + \frac{\pi}{2}$$

$$\frac{\pi}{6} t = \frac{\pi}{2}$$

$$t = 3$$

time to 1st meeting.

$$\frac{2\pi}{3} t = \frac{\pi}{2} t + \frac{\pi}{2} + 2\pi$$

1 LAP

$$\frac{\pi}{2} t + \frac{\pi}{2} + 4\pi + 2\pi$$

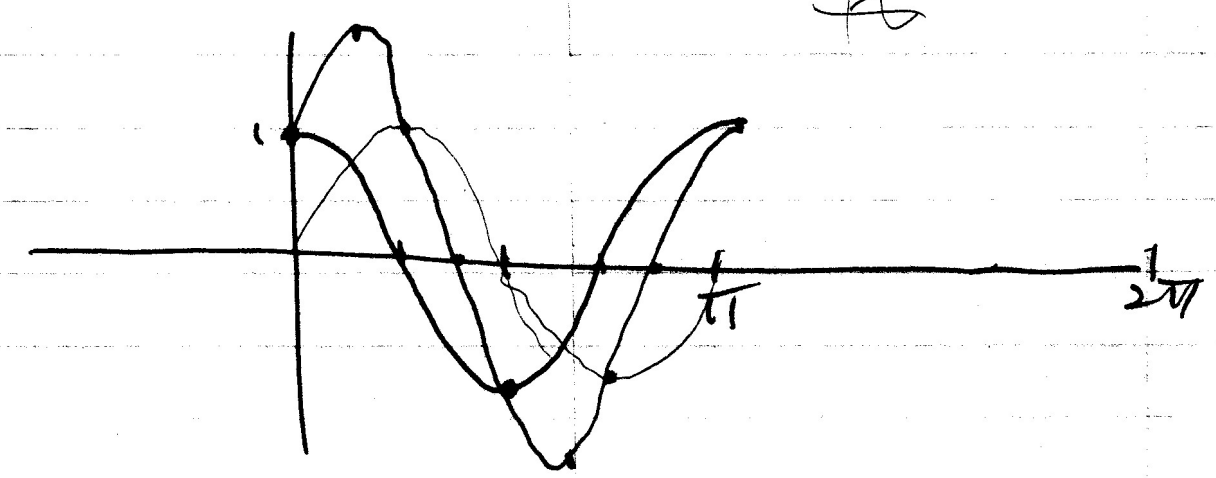
Another LAP.

$$\cos^2 x + 2 \sin x \cos x - \sin^2 x$$

$$\underbrace{\cos^2 x - \sin^2 x} + \underbrace{2 \sin x \cos x}$$

$$\cos 2x + \sin 2x$$

each loop per π.



Honors Trigonometry

No Decimal Answers.

Convince *me* that **you** understand the concept.

Name TRIG Ch3

Per _____ Date 95/99

CHAPTER #3

I This section covers all of the first two pages of this exam. For *each* problem, supply **domain, range, amplitude, period** and **phase shift (including direction)** and **draw** the graph *in the areas provided.* Be sure your drawing crosses or touches the Y-axis. You may do the work for this exam *directly on this* exam paper rather than on the "newsprint" as usual. If you do work on newsprint be sure it is labeled with its problem number. **Indicate significant coordinates.** Each problem in this section is worth 15 points.

use III & ~~IB~~ extra on final

A) $A(x) = 2\cos\left(\frac{2x}{3} + \frac{\pi}{6}\right) + 1$

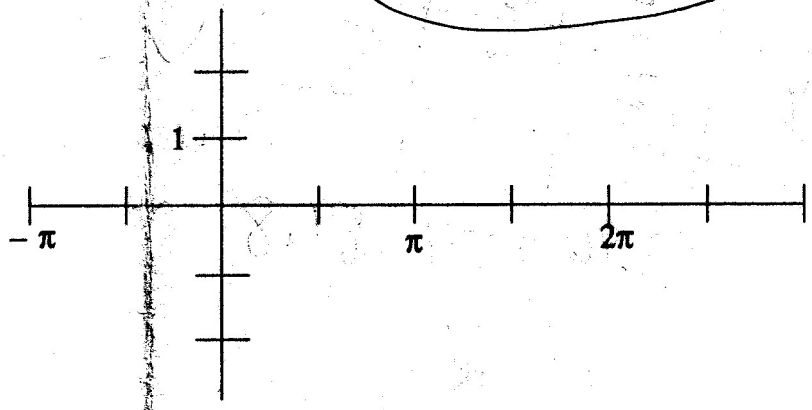
D_A

R_A

Per

Amp

P.S.



B) $B(x) = -2\sin\left(\frac{2x}{3} + \frac{\pi}{2}\right) - 1$

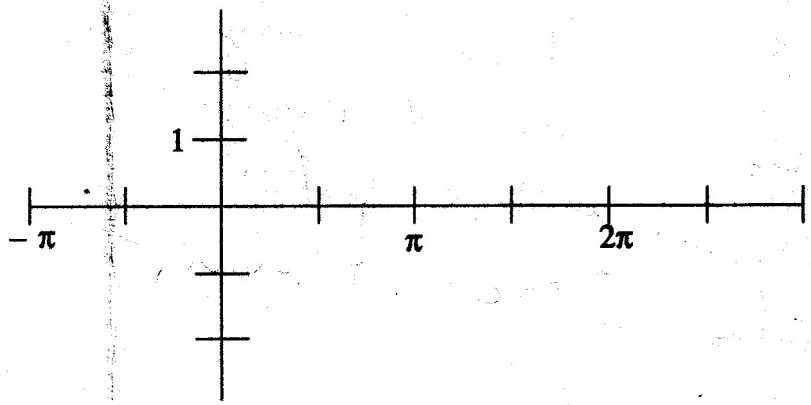
D_B

R_B

Per

Amp

P.S.



C) $C(x) = \csc\left(2x + \frac{\pi}{2}\right) + 1$

D_C $x \neq \frac{(2k-1)\pi}{4}$

R_C $C(x) \leq 0$ or $C(x) \geq 2$

Per π

Amp 1

P.S. $\frac{\pi}{4}$ Left

$2\left(x + \frac{\pi}{4}\right) \overset{\text{Set}}{=} k\pi$

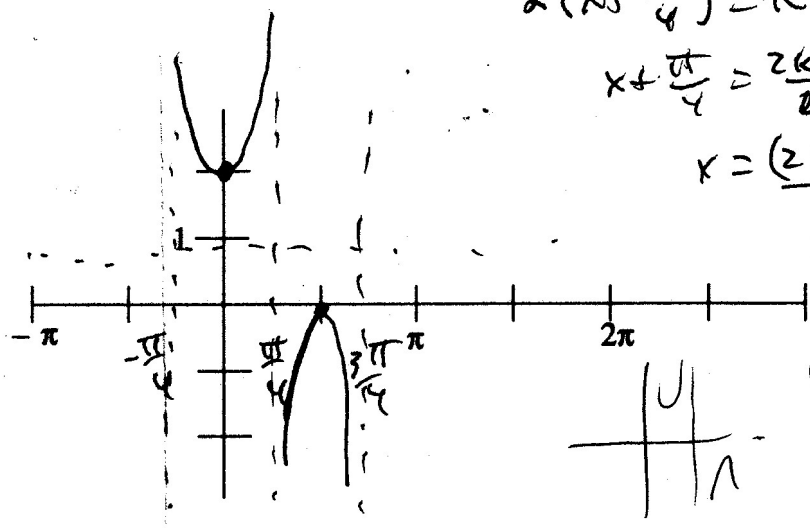
$x + \frac{\pi}{4} = \frac{2k\pi}{4}$

$x = \frac{(2k-1)\pi}{4}$

$k = 0 \quad -\frac{\pi}{4}$

$k = 1 \quad \frac{\pi}{4}$

$k = 2 \quad \frac{3\pi}{4}$



D) $D(x) = \tan\frac{1}{2}x - \frac{1}{2}$

D_D $x \neq (2k+1)\pi$

R_D $D(x) \in \mathbb{R}$

Per 2π

Amp 1

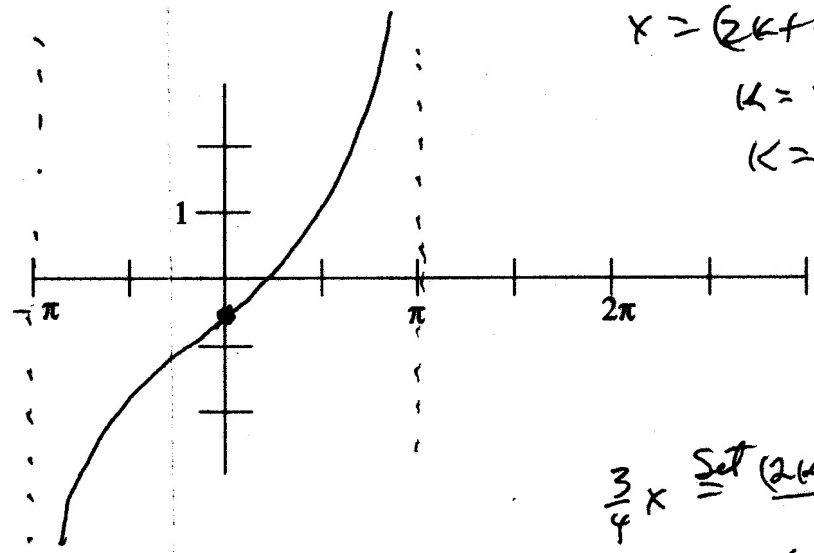
P.S. None

$\frac{1}{2}x \overset{\text{Set}}{=} \frac{(2k+1)\pi}{2}$

$x = (2k+1)\pi$

$k = -1 \quad -\pi$

$k = 0 \quad \pi$



E) $E(x) = \sec\frac{3}{4}x$

D_E $x \neq \left\{\frac{2(2k+1)\pi}{3}\right\}$

R_E $E(x) \leq -1$ or $E(x) \geq 1$

Per $\frac{8\pi}{3}$

Amp 1

P.S. None

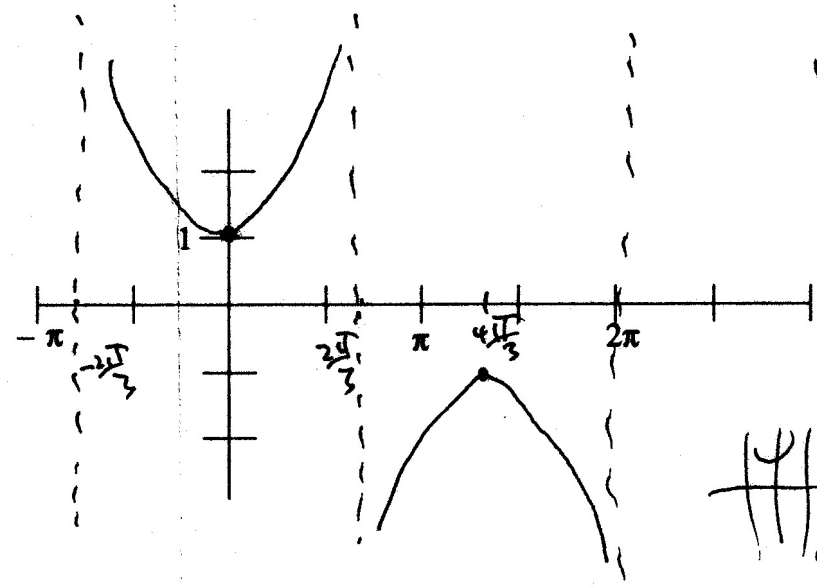
$\frac{3}{4}x \overset{\text{Set}}{=} \frac{(2k+1)\pi}{2}$

$\frac{2(2k+1)\pi}{3}$

$k = -1 \quad -\frac{2\pi}{3}$

$k = 0 \quad \frac{2\pi}{3}$

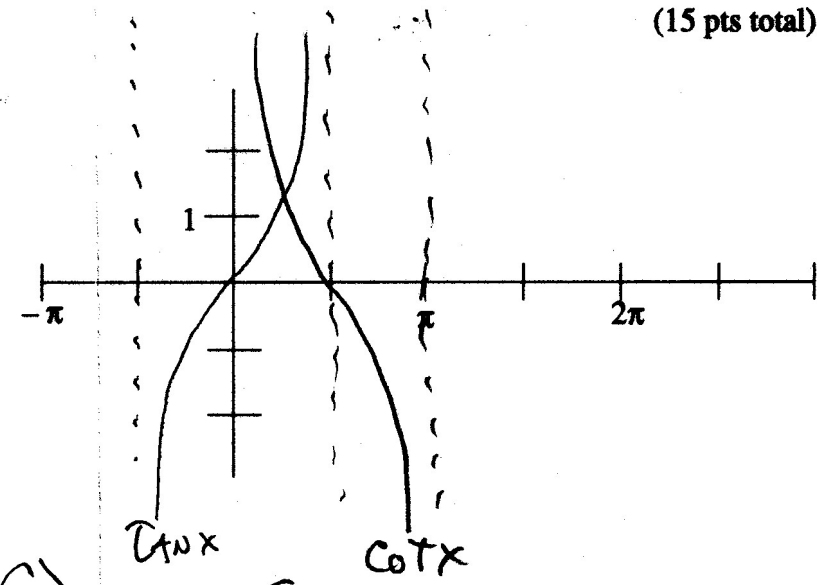
$k = 1 \quad 2\pi$



II Draw a sketch of tan x and cot x on the supplied axis. (15 pts total)

Describe what must be done to tan x to make it coincide with cot x.

Prove that your changes would have the desired effect.



make Tan x negative then shift it $\frac{\pi}{2}$ to Right.

$-\tan\left(x-\frac{\pi}{2}\right)$

$-\dfrac{\sin\left(x-\frac{\pi}{2}\right)}{\cos\left(x-\frac{\pi}{2}\right)}$

$-\dfrac{\sin x\cos\frac{\pi}{2}-\sin\frac{\pi}{2}\cos x}{\cos x\cos\frac{\pi}{2}+\sin x\sin\frac{\pi}{2}}$

$-\dfrac{0-\cos x}{0+\sin x}$

$+\cot x$

III Given P_1 starts at $\left(\frac{-1}{2},\frac{\sqrt{3}}{2}\right)$ with $\omega_1=\frac{\pi}{4}$ and P_2 starts at $\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$ with $\omega_2=\frac{\pi}{3}$

(10 pts tot)

A) Find t when the two points first concide.

$\frac{\pi}{4}t+\frac{5\pi}{12}=\frac{\pi}{3}t$ $\quad t=5$

$\frac{5\pi}{12}=\frac{\pi}{12}t$

$\frac{2\pi}{2}-\frac{\pi}{4}$

$\frac{5\pi-3\pi}{12}$

B) Confirm that P_1 and P_2 do, in fact, have the same coordinates at the t found in part A.

$\cos\left(\frac{\pi}{4}\cdot 5+\frac{2\pi}{3}\right),\ \sin(\quad)$

$\cos\left(\frac{5\pi}{4}+\frac{2\pi}{3}\right)$

$\cos\frac{(15+8)\pi}{12}$

$\cos\frac{23\pi}{12},\ \sin\frac{23\pi}{12}$

$=$

$\cos\left(\frac{\pi}{3}\cdot 5+\frac{\pi}{4}\right),\ \sin(\quad)$

$\cos\frac{(20+3)\pi}{12}$

$\left(\cos\frac{23\pi}{12},\ \sin\frac{23\pi}{12}\right)$

EXTRA CREDIT ---------- **5 pts** ----------

Given $f(x)=\cos 2x\sin x+\sin 2x\cos x$. Prove $f(x)$ has period $\frac{2\pi}{3}$.

$=\sin 2x\cos x+\sin x\cos 2x$

$=\sin(2x+x)$

$f(x)=\sin 3x$

$f\left(x+\frac{2\pi}{3}\right)=\sin 3\left(x+\frac{2\pi}{3}\right)$

$=\sin(3x+2\pi)$

$=\sin 3x$

$=f(x)$

$\therefore f$ has period $\frac{2\pi}{3}$.

1995

	5	6	T
100	1	1	2
90	15	4	19
80	6	14	20
70	1	6	7
60	2	0	2
54	2	3	5
	83.96	79.71	81.8

104

41

27 28

10/29/99

	4	5	T
100	0	0	0
90	4	11	15
80	11	7	18
70	7	5	12
60	2	0	2
54	2	0	2
	2070/26	2007/23	4077/49
	79.6	87.3	83.2

98

Just another Great exam.

I should allow decimal coordinates to III B. It won't make any difference except those with calcs get a chance to use them.

Honors Trigonometry

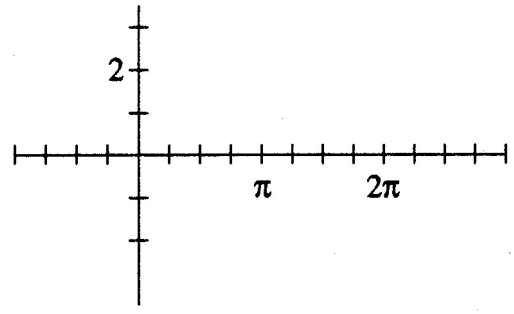
Name _____

El Quizzo

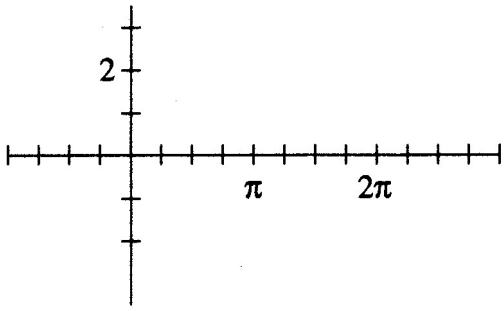
Per _____ Date _____

For each of the following, state Range, Domain, Period, Phase shift (including direction) and amplitude. Graph on the supplied axis.

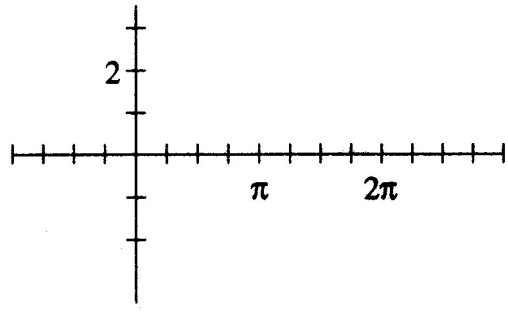
1) $F(x) = \sec\left(\frac{2}{3}x + \frac{\pi}{6}\right) + 1$



2) $F(x) = \tan\left(\frac{2}{3}x + \frac{\pi}{6}\right) - 1$



3) $F(x) = 2\cos\left(\frac{2}{3}x + \frac{\pi}{6}\right)$



Honors Trigonometry

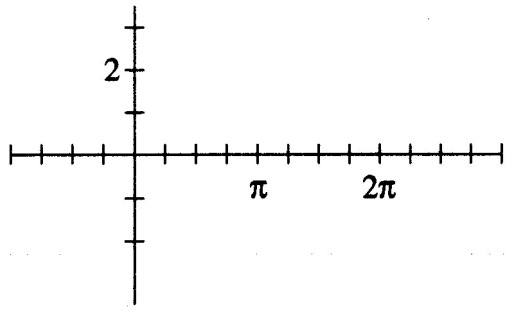
El Quizzo

Name _____

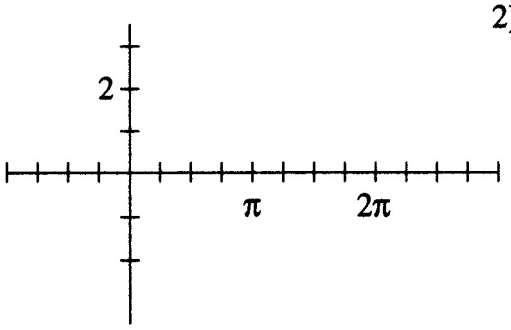
Per _____ Date _____

For each of the following, state Range, Domain, Period, Phase shift (including direction) and amplitude. Graph on the supplied axis.

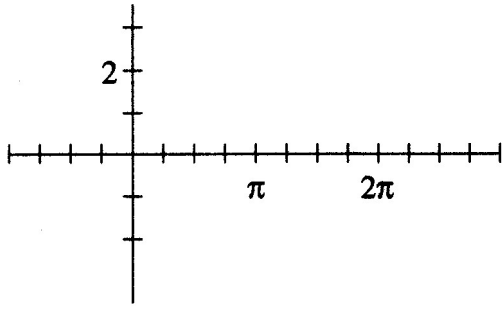
1) $F(x) = \csc\left(\frac{2}{3}x + \frac{\pi}{6}\right) - 1$



2) $F(x) = \cot\left(\frac{2}{3}x + \frac{\pi}{6}\right) + 1$



3) $F(x) = 2\sin\left(\frac{2}{3}x + \frac{\pi}{6}\right)$



Jake's Graphing Adventure

I

For each of the following problems:

1. Draw an axis and sketch the graph of the function .
   Remember to show:
   A) how you compute the new domain restrictions or the new zeros
   B) the proper generic equation.
2. State the
   A) Domain
   B) Range
   C) Period
   D) Phase shift (including a direction word like 'left')

a) $f(x) = 2\sin\left(\frac{4x}{3} - \frac{\pi}{3}\right) - 1$

b) $f(x) = -2\cos\left(\frac{3x}{2} - \pi\right) + 2$

c) $f(x) = -\tan\left(\frac{x}{2} - \frac{\pi}{6}\right) - 2$

d) $f(x) = \csc\left(\frac{2x}{3} - \frac{\pi}{6}\right) + 1$

e) $f(x) = \sec\left(\frac{4x}{3} - \frac{8\pi}{9}\right)$

f) $f(x) = \cot\left(\frac{3x}{2} - \frac{\pi}{4}\right) + 1$

g) $f(x) = -\sin\left(\frac{x}{2} + \frac{\pi}{6}\right) + 1$

h) $f(x) = \cos\left(\frac{2x}{3} - \frac{\pi}{6}\right) - 1$

i) $f(x) = 2\tan\left(\frac{4x}{3} - \frac{2\pi}{3}\right) - 2$

j) $f(x) = -2\csc\left(\frac{3x}{2} - \frac{\pi}{4}\right) + 1$

k) $f(x) = \sec\left(\frac{x}{2} + \frac{\pi}{4}\right) - 1$

l) $f(x) = \cot\left(\frac{2x}{3} + \frac{2\pi}{9}\right) + 3$

m) $f(x) = -3\cos\left(\frac{4x}{3} - \frac{2\pi}{9}\right)$

n) $f(x) = \tan\left(\frac{3x}{2} + \frac{3\pi}{4}\right) + 1$

o) $f(x) = 3\sin\left(\frac{2x}{3} + \frac{4\pi}{9}\right)$

p) $f(x) = 2\csc\left(\frac{x}{2} + \frac{\pi}{6}\right) - 1$

q) $f(x) = \sec\left(\frac{2x}{3} - \frac{\pi}{6}\right) + 1$

r) $f(x) = \cot\left(\frac{4x}{3} - \frac{\pi}{6}\right)$

II

For each set of functions, Sketch using different colors for each function. For each problem, locate 2 x-values on your graph and show the coordinates for each of the functions at those x-values.

a) $f(x) = \sin 2x$, $g(x) = \sin x$, $h(x) = f(x) + g(x)$

b) $f(x) = \cos 2x$, $g(x) = \sin x$, $h(x) = f(x) + g(x)$

c) $f(x) = \cos 2x$, $g(x) = \cos x$, $h(x) = f(x) + g(x)$

d) $f(x) = \sin 2x$, $g(x) = \cos x$, $h(x) = f(x) + g(x)$

e) $f(x) = \sin 2x$, $g(x) = -\cos x$, $h(x) = f(x) + g(x)$

f) $f(x) = 2\cos x$, $g(x) = \cos x$, $h(x) = f(x) + g(x)$

Honors Trigonometry

Copy original problem.

Convince *me* that **you** understand the concept!

Name Trig

Per _____ Date ~~Ch 3~~ MAKEUP

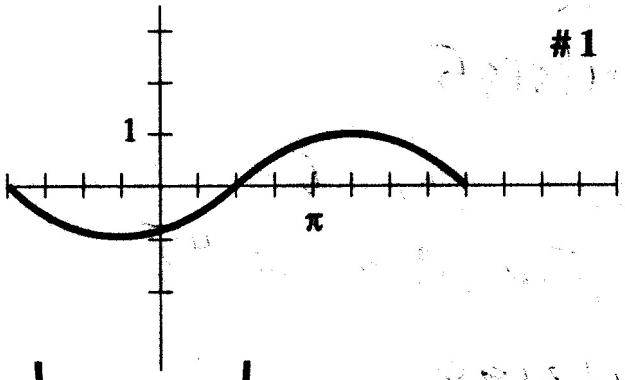
Chapter 3 *Make–Up* Exam

I Consider figure #1. Determine A, B, C, D, the period length, phase shift (Note: phase shift cannot exceed period length.) domain and range. (10 pts ea)

a) $A > 0;\quad A(x) = A\sin(Bx - C) + D$

b) $A < 0;\quad A(x) = A\sin(Bx - C) + D$

c) $A > 0;\quad A(x) = A\cos(Bx - C) + D$

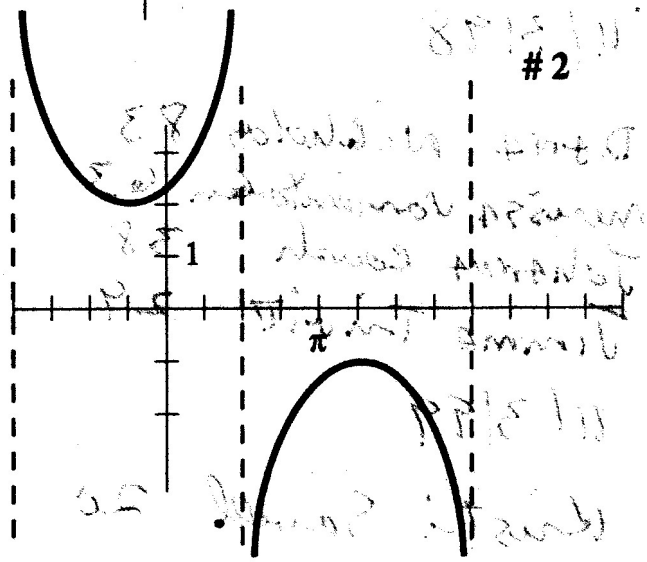


II Consider figure # 2. Determine A, B, C, D, the period length, phase shift (Note: phase shift cannot exceed period length.) domain and range. (10 pts ea)

a) $A > 0;\quad A(x) = A\sec(Bx - C) + D$

b) $A < 0;\quad A(x) = A\sec(Bx - C) + D$

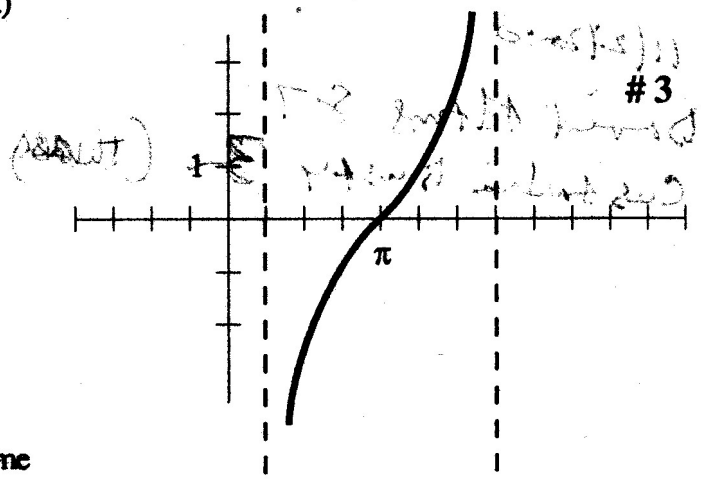
c) $A > 0;\quad A(x) = A\csc(Bx - C) + D$



III Consider figure # 3. Determine A, B, C, D, the period length, phase shift (Note: phase shift cannot exceed period length.) domain and range. (10 pts ea)

a) $A > 0;\quad A(x) = A\tan(Bx - C) + D$

b) $A < 0;\quad A(x) = A\cot(Bx - C) + D$



IV Given point A starting at (3 , 0) with $\omega_A = -\frac{\pi}{3}$ and point B starting at $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$ with $\omega_B = -\frac{\pi}{4}$. When points A and B conincide, obviously they are at the same location. Show that, in fact, you do get the same coordinates using ω_A and then using ω_B. How far does *each* point travel from time = zero until the points coincide? (20 pts)

I $A(x) = A\sin(Bx - C) + D \quad A > 0$

a) $A(x) = 1\sin B(x - \frac{C}{B}) + D$

$\sin \frac{2}{3}(x - \frac{\pi}{2}) + 0$

$\sin(\frac{2}{3}x - \frac{\pi}{3})$

$A = 1$

$B = \frac{2}{3}$ D_A $x \in \mathbb{R}$

$C = \frac{\pi}{3}$ R_A $-1 \le A(x) \le 1$

$D = 0$ Per 3π

Ph Shift $\frac{\pi}{2}$ Right

b) $A(x) = A\sin B(x - \frac{C}{B}) + d$

$A(x) = -1\sin \frac{2}{3}(x + \pi) + 0$

$= -\sin(\frac{2}{3}x + \frac{2\pi}{3}) + 0$

$A = -1$ D_A $x \in \mathbb{R}$

$B = \frac{2}{3}$ R_A $-1 \le A(x) \le 1$

$C = -\frac{2\pi}{3}$ Per 3π

$D = 0$ PS π Left

c) $A(x) = A\cos B(x - \frac{C}{B}) + D$

$1\cos \frac{2}{3}(x - \frac{5\pi}{4}) + 0$

$\cos(\frac{2}{3}x - \frac{5\pi}{6})$

$A = 1$

$B = \frac{2}{3}$ D_A $x \in \mathbb{R}$

$C = \frac{5\pi}{6}$ R_A $-1 \le A(x) \le 1$

$D = 0$ Per 3π

PS $\frac{5\pi}{4}$ Right

II a $A\sec B(x + \frac{C}{B}) + d$

$= \frac{3}{2}\sec \frac{2}{3}(x + \frac{\pi}{4}) + \frac{1}{2}$

$\frac{3}{2}\sec(\frac{2}{3}x + \frac{\pi}{6}) + \frac{1}{2}$

$A = \frac{3}{2}$

$B = \frac{2}{3}$ D_A $x \neq \{\frac{(3k+1)\pi}{2}\} \frac{\pi}{2} + \frac{3k\pi}{2}$

$C = -\frac{\pi}{6}$ R_A $A(x) \le -1$ or $A(x) \ge 2$

$D = \frac{1}{2}$ Per 3π P.S. $\frac{\pi}{4}$ Left

II b $A\sec B(x + \frac{C}{B}) + d$

$-\frac{3}{2}\sec \frac{2}{3}(x - \frac{5\pi}{4}) + \frac{1}{2}$

$-\frac{3}{2}\sec(\frac{2}{3}x - \frac{5\pi}{6}) + \frac{1}{2}$

$A = -\frac{3}{2}$

$B = \frac{2}{3}$ D_A $x \neq \{\frac{(3k+1)\pi}{2}\}$

$C = -\frac{5\pi}{6}$ R_A $A(x) \le -1$ or $A(x) \ge 2$

$D = \frac{1}{2}$ Per 3π P.S. $\frac{5\pi}{4}$ Right.

II c $A(x) = A\csc B(x - \frac{C}{B}) + d$

$= \frac{3}{2}\csc \frac{2}{3}(x + \pi) + \frac{1}{2}$

$\frac{3}{2}\csc(\frac{2}{3}x + \frac{2\pi}{3}) + \frac{1}{2}$

$A = \frac{3}{2}$

$B = \frac{2}{3}$ D_A $x \neq \{\frac{(3k+1)\pi}{2}\}$

$C = -\frac{2\pi}{3}$ R_A $A(x) \le -1$ or $A(x) \ge 2$

$D = \frac{1}{2}$ Per 3π P.S. π Left

III

A) $A(x) = A\tan B(x - \frac{C}{B}) + D$

$= 1\tan \frac{2}{3}(x - \pi) + 0$

$\tan(\frac{2}{3}x - \frac{2\pi}{3})$

$A = 1$

$B = \frac{2}{3}$

$C = \frac{2\pi}{3}$

$D = 0$

Per $\frac{3\pi}{2}$ PS. π Right

D_A $x \neq \{\frac{(6k+1)\pi}{4}\}$

$\frac{\pi}{4} + \frac{6k\pi}{4}$

$\frac{(6k+1)\pi}{4}$

R_A $A(x) \in \mathbb{R}$

Per $\frac{3\pi}{2}$, PS. π Right

B) $A(x) = A\cot B(x - \frac{C}{B}) + D$

$= -\cot \frac{2}{3}(x - \frac{\pi}{6}) + 0$

$-\cot(\frac{2}{3}x - \frac{\pi}{6})$

$A = -1$

$B = \frac{2}{3}$

$C = \frac{\pi}{6}$

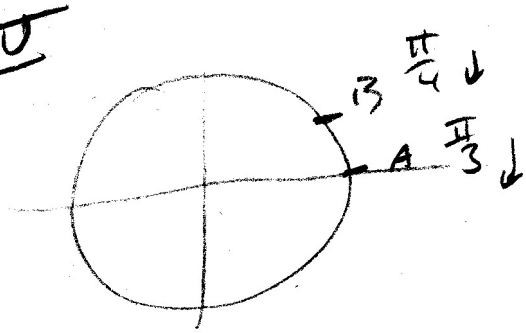
$D = 0$

$x \neq \{\frac{(6k+1)\pi}{4}\}$

R $A(x) \in \mathbb{R}$

Per $\frac{3\pi}{2}$ PS $\frac{\pi}{4}$ Right

IV



$3\frac{\pi}{3}t = 3\cdot\frac{\pi}{4}t + 3\cdot\frac{11\pi}{6}$

$\pi t = \frac{3\pi t}{4} + \frac{11\pi}{2}$

$\frac{1}{4}t = \frac{11}{2}$

$t = 22$

$A = 3\frac{\pi}{3}\cdot 22 = 22\pi$

$B = 3\cdot\frac{\pi}{4}\cdot\frac{11}{2} = \frac{33\pi}{2}$,

$(p\cos(\omega t + b)$, etc)

$(3\cos(-\frac{\pi}{3}\cdot 22)$,

$(3\cos(-\frac{22\pi}{3})$,

$(3\cos(\frac{2\pi}{3})$,

$(-\frac{3}{2}, \frac{3\sqrt{3}}{2})$

$3\cos(-\frac{\pi}{4}\cdot 22 + \frac{\pi}{6})$,

$3\cos(-\frac{11\pi}{2} + \frac{\pi}{6})$

$3\cos(-\frac{32\pi}{6})$

$3\cos(+\frac{36\pi}{6} - \frac{32\pi}{6})$

$3\cos(\frac{4\pi}{6})$

$3\cos(\frac{2\pi}{3})$, etc.

$(-\frac{3}{2}, \frac{3\sqrt{3}}{2})$

11/10/93

Jaime Sollee 85

Brent Posey 47

Alan Hafez 73

Nick Baldwin 85

11/8/95

Melissa Villa 6

Crystal Patkowski 45

11/3/98

Dana Nicholas 83

Melissa Vandritsken 63

Johanna Coash 38

Jimma Trivitt 29

11/3/99

Kristi Sawell 20

11/2/2000

David Hang 87

Cassandra Busby 3 (three)

Chapter 3
HONORS TRIGONOMETRY

T E A C U P S

Mr. Mumaugh

I A car wheel is rotating at 900 revolutions per minute.

A) Determine the angular velocity in radians per second on the tire tread, 35 cm from the center.
B) Determine the angular velocity in radians per second on the rim between the tire and the wheel, 20 cm from the center.
C) Find the following : The linear velocity in meters per second ...
   1) on the tire tread.
   2) on the rim between the tire and the wheel.
   3) at the center.
D) Determine the car's speed in km per hour. $\approx$ 119 kph

II A salt shaker is sitting 4 cm from the center of a lazy-susan. If the lazy-susan is turned through 120 ° ...

A) Through how many radians does the shaker turn?
B) What distance does the shaker travel?
C) If the lazy-susan was turned that far in one-half second, what was the shakers angular velocity?
D) What was the shaker's linear velocity?

III Bull Dawg is riding on a ferris wheel. He notices that he is turning three revolutions per minute and that his seat is twenty-five feet from the axle of the wheel.

A) What is his angular velocity in revolutions per minute?
B) What is his angular velocity in degrees per minute?
C) What is his angular velocity in radians per minute?
D) What is his linear velocity in radians per minute?

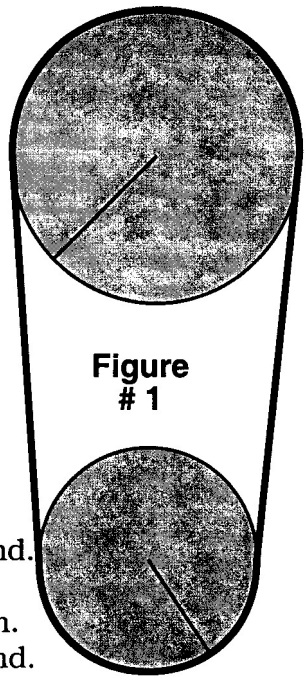


Figure # 1

IV A small pulley 6 cm in diameter is connected by a belt to a larger pulley 15 cm in diameter, as shown in figure # 1.

A) The small pulley is driving at 120 rpm.
   1) Find the angular velocity of the drive pulley in radians per second.
   2) Find the linear velocity of the rim of the drive pulley.
   3) What is the linear velocity of the rim of the slave pulley? Explain.
   4) Find the angular velocity of the slave pulley in radians per second.
   5) How fast is the slave pulley turning in revolutions per minute?

B) The large pulley is driving at 120 rpm. ---- Answer the same 5 questions in part A.

V A large pulley has diameter 30 cm. The small pulley has a diameter of 8 cm.

A) Answer the same questions in IV A.

B) Answer the same questions in IV B.

VI

A water wheel, 14 meters in diameter is rotating as shown is figure # 3. You start a stop watch and observe the motion of point P on the rim of the wheel. When the stopwatch reads 3 seconds, P is at its maximum distance from the water. When the watch reads 8 seconds, P is at its maximum depth below the water.

A) Find the angular velocity of P in rpm AND in radians per second.

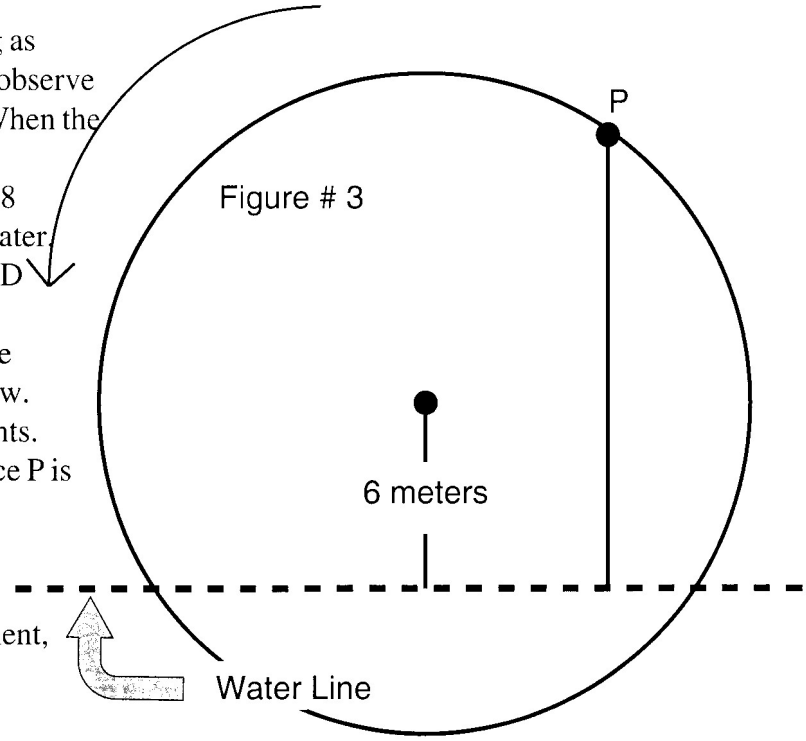
B) The distance of P from the water's surface varies with time. Plot the points you know. Sketch a cosine function through the points.

C) Write the equation expressing the distance P is from the water in terms of seconds t.

D) The general sinusoidal (look it up) equation is: $f(x) = a\cos(bx - c) + d$.

You know that A is amplitude, D is axis movement, and the fraction $\frac{c}{b}$ is phase shift. Up till now, you have learned only that "b" helps you determine what the period is. ***Now you know the real answer!***

Explain what "*b*" *really* is.



VII

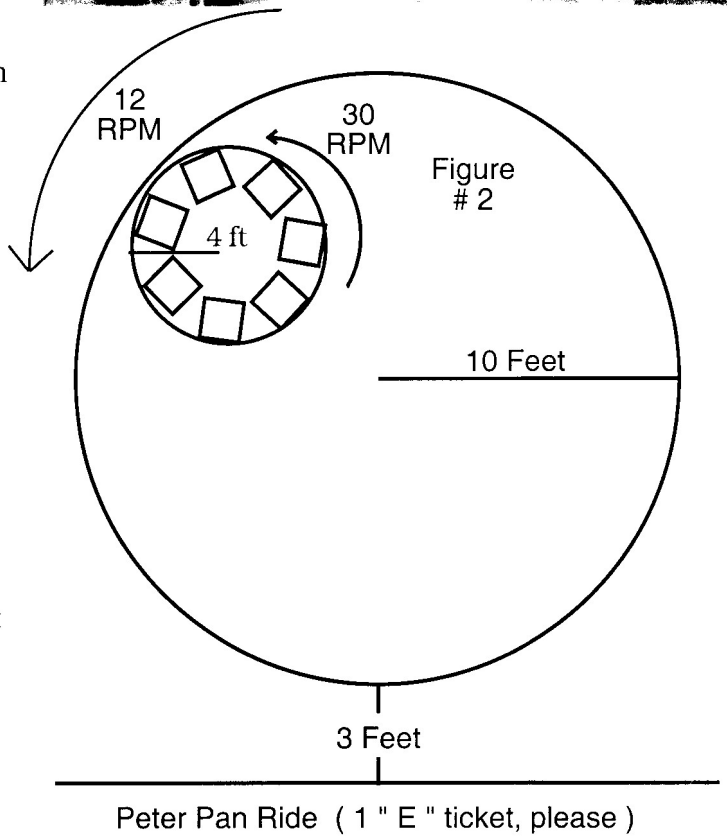
One of the rides in Disneyland is called TEACUPS. It is a merry-go-round that rotates at a constant angular velocity while rings of seats rotate at a different (but constant) angular velocity. Suppose that the seats rotate at 30 rpm and the merry-go-round rotates at 12 rpm.

A) Find your linear velocity in feet per second due to the combined rotations of both wheels when your seat is:
   1. farthest from the center of the merry-go-round.
   2. nearest to the center of the merry-go-round.

B) In what direction are you actually moving when your seat is closest to the center of the merry-go-round?

C) As your ride continues, your distance from the Peter Pan ride varies. Suppose your seat is 23 feet from the fence at $t = 0$ seconds. Write an equation expressing your distance from the fence in terms of t.

D) Sketch your function.



VIII The Courtship of Jake (the Snake) Jakester and the Czarina

In order to fully understand the particulars of Jake and the Czarina's love life, certain definitions need to be made:

- The Czarina's affection for (i.e. *interest in*) Jake is characterized by: $C(t) = \sin\left(\frac{9\pi t}{10}\right)$
- Jake's affection for the Czarina is characterized by: $J(t) = 2\sin\left(\frac{7\pi t}{10}\right)$
- The time t is measured in weeks.
- The Czarina and Jake met at $t = 0$.
- You "like" someone when you have a positive affection for him or her.
- You "dislike" someone when you have a negative affection for him or her.
- Their feelings for each other "change" at a time $t = a$, the first time that Jake and the Czarina have no affection for each other and the interest of each in the other is waning.
- A couple is "happy" when the both like each other.
- A couple is "unhappy" when they both dislike each other.
- An individual is "happy" when he or she is liked by another.
- An individual is "unhappy" when he or she is disliked by another.

a) Who has the greater emotional range, Jake or the Czarina? Explain.

b) Whose emotions fluctuate more rapidly?

c) Graph Jake and the Czarina's first 3 months on the same axis.

d) When was the first time that Jake and the Czarina felt indifference for each other, but both their affections for each other were on the increase? When was the second time?

e) On a day-to-day basis, describe Jake and the Czarina's first two weeks (i.e. how they felt about each other).

f) Using your graphs, describe what was "happening" at $t = 10$ weeks.

g) When does the Czarina like Jake intensely while Jake dislikes The Czarina intensely?

h) Describe how you can find, via the graphs, when Jake and the Czarina were happy as :

   1) individuals

   2) a couple

Precalculus*
Spring 2002

Name_____
Per____Date_____
DUE Friday April 26, 2002

TRIG PROJECT

READ and FOLLOW these directions carefully. Points will be deducted if directions are not followed carefully.

1. Retype each problem on a word processor before working it. Then show all work on the problem. The work may be shown by hand, following the guidelines below. (or lose points) You may re-word problems to create your own story, but keep the mathematical content the same as in the problem statement.

2. Put only one problem on a page with problems on only one side of a page. Be sure to leave a wide enough left margin for the type of folder you are using. (or lose points)

3. Include a title page and a binder or report cover similar to what you would use for an English paper. Arrange problems in the order given when handing in your project. You may use a folder with fasteners in it. However, a plain manila file folder or pocket folder is not acceptable. (or lose points)

4. Draw a diagram for each problem. Attempt to make diagrams as nearly to scale as possible. This may make problems easier to solve. Be sure to label all quantities represented in the diagram whether they are knowns or unknowns. (or lose points)

5. Show all steps in your solution in a neat and organized manner. Use a format which makes work easier to follow such as the formula - substitution - solution format which you have learned. I need to be able to follow all of your work. This includes how you got your angles, even if just "doing the 180-thing" from Geometry. Do not leave anything to my imagination. (or lose points)

6. Use the following guidelines for rounding answers unless stated differently in a problem. You may round intermediate answers to four (4) decimal places or keep all decimals. In final answers round lengths to the nearest hundredth unless stated otherwise. If the angle(s) in a problem are given in degrees, minutes, and seconds (DMS), give your answer(s) for other angle(s) in DMS also. Otherwise, round to the nearest hundredth of a degree. NOTE: If the answer to one part of a problem must be used to answer a subsequent part, you should use the UNROUNDED intermediate answer or a 4-place decimal intermediate answer. The safest thing to use the STO button on the calculator to keep all the decimals. Points will be deducted EACH time these guidelines are not followed. Your answers should all match mine. (or lose points)

7. Be sure answers are indicated or marked clearly. This includes important intermediate answers. I must be able to find everything!

8. Label all answers with appropriate units such as degrees, DMS, feet, miles, knots, etc. as dictated by the problem if they are given. If you need to give a heading or bearing, you may use either notation which we used, as long as your numbers are correct.

9. Points will be included in the grade for following directions, rounding/accuracy, neatness, thoroughness, creativity/uniqueness, and appropriateness of methods used.

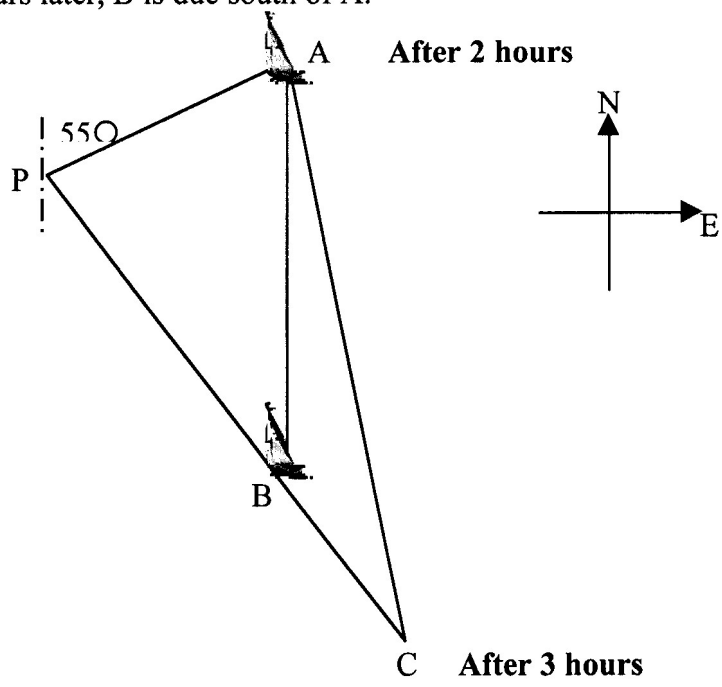
10. You are to do all of the following problems.

11. You must include a brief description or list of names of people you worked with or received HELP from on EACH PROBLEM. If you did not receive any help on a problem, tell me that also. There is to be no copying of problems, but you may HELP one another. Any deviation from this will be severely penalized. Also include the names of any teachers or any other non-students who helped.

THE PROBLEMS

Note: Problems 1 and 2 are from a placement test for Oxford and Cambridge Universities in England. They were provided by former Hoover Student Chris Prengaman.

1. **Gone Fishin'** Two small fishing boats leave a harbour (point P in the diagram) at the same time. The first boat, A, heads on a bearing of 055O at a steady speed of 9 kilometres per hour whilst the second boat, B, leaves on a bearing of 135O, at a steady speed of "v" kilometres per hour. Two hours later, B is due south of A.



A) Copy the diagram and mark in the known angles and the length PA.

B) Calculate the distance PB and hence give the speed of the boat B, giving your answer correct to two decimal places. (NOTE: 2 answers here)

C) Boat A ancors (i.e. stops) whilst B continues in the same direction as before for another hour, to point C. How far apart are the two boats now? Give your answer correct to two decimal places.

D) If the skipper of Boat B (at location C) decides to travel to Boat A, what will be the proper heading that must be taken? (nearest TENTH of a degree) (Note: This part added by Mr. K)

2. **Message in an . . . Oil Drum??** Whilst walking along cliffs in a northerly direction, Lestor notices an oil drum floating in a calm sea, gradually drifting away on the current. The cliffs are vertical and 20 metres high; the ground at the top of the cliffs is level.

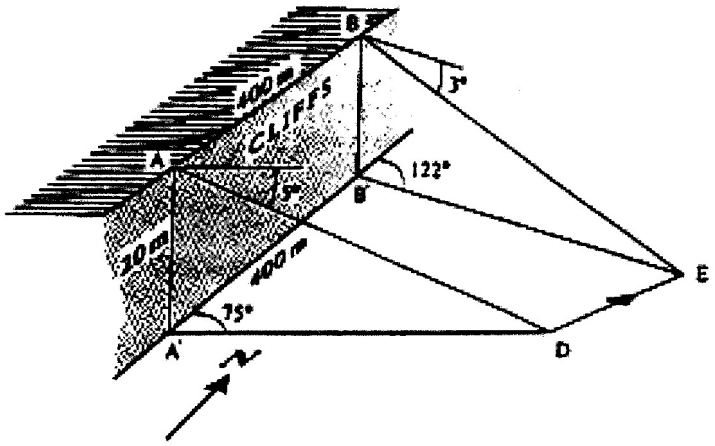
To start with, Lestor is at point A, seeing the oil drum at point D on a bearing of 075O with an angle of depression of 5O. Listor then walks 400 metres due North in 5 minutes to B. The oil drum is then at point E on a bearing of 122O with an angle of depression of 3O. The point A′ is at the base of the cliffs vertically below A and similarly B′ is vertically below B.

A) Show that length A′D = 228.6 metres and calculate length B′E.

B) Calculate length B′D in triangle A′B′D.

C) Calculate angle A′B′D and hence find angle DB′E.

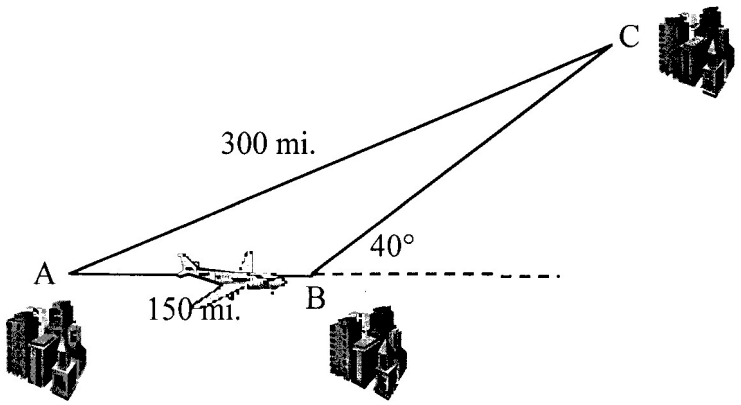
D) Calculate the speed of the drum in metres per second. (note, this will require at least one additional intermediate calculation)



3. **Who's Navigating Anyway!?** An airplane flies from City A to City B, a distance of 150 miles, then turns through an angle of 40° and heads toward city C, as shown in the figure.

   A) If the distance between cities A and C is 300 miles, how far is it from City B to City C?

   B) Through what angle should the pilot turn (the smallest possible) at City C to return to City A. (Note, this is different that the plane's compass heading, which you do not need to find.)



4. The drive wheel of an engine is 13 inches in diameter, and the pulley on the rotary pump is 5 inches in diameter. If the shafts of the drive wheel and the pulley are 2 feet apart, answer each of the following.

   A) What length of belt is required to join the wheel and the engine and the pulley as shown in the figure below at the LEFT? NOTE: Alternate drawings at end of document!

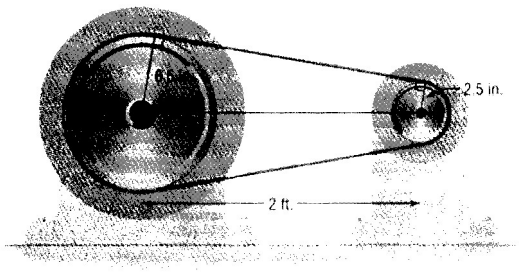


Figure for parts A, B, C

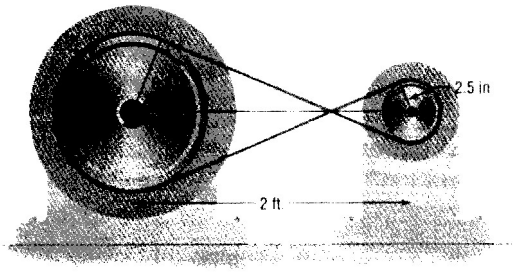


Figure for part D

   B) If the drive wheel spins at a rate of 25 rpm, what is the linear velocity of the belt? Include appropriate units in your answer (you decide what to use!)

   C) What will be the angular velocity of the pulley on the pump (in rpm)?

Honors Trig
Chapter 3
Mr. Mumaugh

WEEKEND QUICKIE

Horatio Hypochondria has been absent from school so long that even he has forgotten where his desk is in the classroom. Horatio called ***you*** about 6:30 P.M. (Horatio, as we all know, has more brass than a band.) and begged you to "catch him up" on what he has missed in your favorite class, **TRIGONOMETRY**. Horatio seems to have forgotten everything! You start to panic and then remember that *your* awesome trig expertise will save the day for Horatio. ***You can*** change Horatio from a lowly pretender to a worthy contender! You quickly jot down the following list of topics.
You will turn in your written (and very explicit answers) to the following questions. Include drawings when they will help to clarify a point you are making but remember that pictures are not to take the place of explanations! This homework is worth 5 regular assignments. ***Only the very best and most complete responses should expect pluses.***

I A) Explain *everything there is to know* about $a, b, c,$ and d in : $\boldsymbol{a}$ **Fctn** ($\boldsymbol{bx - c}$) $+ \boldsymbol{d}$.

B) Explain how knowing which intervals on the x-axis correspond to which quadrants on the unit circle helps in drawing the graph of *any* trig function. Explain what the "argument" is. Explain how we determine the NEW period length.

C) Explain what a "generic" picture is and how we use it. Explain how the "asymptote method" and the "zeros method" are similar and how they are different and when you and why you would use each to assist in drawing the graph of a trig function.

D) Explain, in words, how to properly graph the function: $A(x) = 2\sin\left(\frac{2}{3}x - \frac{\pi}{6}\right)$

E) Draw the graph of $f(x)$. Be sure your method is VERY clear to Horatio (and me). On each picture, select 1 point on the graph which is not on an axis and estimate the (x, y) coordinate.
Compute $f($ of that x-value). Your estimate and your computed value should be very close.

1) $f(x) = 2\sin x + \cos x$ 2) $f(x) = 2\sin x \cos x$ 3) $f(x) = \sqrt{\sin^2 4x + \cos^2 4x}$

4) $f(x) = \cos\frac{1}{2}x + 2\sin\frac{3}{2}x$ 5) $f(x) = \csc^2 x - \cot^2 x$

II A) Explain what "omega" is. Explain what " rho " is.

B) Explain the difference between rotational , linear, and angular velocities.

C) Explain ***EVERYTHING*** there is to know about two points traveling around a non-unit circle at different speeds (and directions) .

D) p_1 starts at (3, 0) and p_2 starts at $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$, $\omega_1 = -\frac{\pi}{3}$ and $\omega_2 = -\frac{\pi}{4}$.

1) When will the two points coincide for the first time after $t = 0$? Use ω_1 to compute the coordinates for that first meeting. Next use ω_2 to compute the coordinates. They are the same point aren't they?

2) When AND where will the two points coincide for the **third** time after $t = 0$?

III Close your phone call from Horatio with a "Thank you." because now **you** are ready for your 3 trig exam.